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Problem 43 Extragalactic cosmic-ray detection at ground level

The Nishimura-Kamata-Greisen (NKG) formula provides a good estimate of the number density of the secondary particles in an Extensive Air Shower as function of the distance to the shower axis and the shower development stage.

The formula differs slightly for different kind of secondary particles, and for the electromagnetic component (i.e. $e^{\pm}$) can be written as

$$\rho_{em}(r, Ne) = \frac{0.4\ Ne}{r_M^2} \left(\frac{r_M}{r}\right)^{2-s} \left(\frac{r_M}{r + r_M}\right)^{4.5-s} \left(1 + \frac{r}{11.4\ r_M}\right),$$

where Ne is the total amount of electrons in the shower, r_M is the Molière radius (approximately equal to 0.25 X_0, $X_0 = 36.66$ g cm^{-2} or 300 m at sea level), r is the distance from the shower axis, and $s = (3t)/(t + 2t_{max})$ is the shower age parameter (where t is the atmospheric depth in terms of radiation lengths X_0).

a) Asume an air shower, initiated by a proton, impinging vertically onto the atmosphere with an energy of $5 \cdot 10^{17}$ eV and write down the NKG formula for an observer located at the Pierre Auger Observatory (i.e. $X = 870$ g cm^{-2}).
Hint: use the Heitler model to estimate N_{max}, X_{max}, and the age parameter s. Use the Gaisser-Hillas formula (see also Problem 30)

$$N(X) = N_{max} \left(\frac{X - X_{first}}{X_{max} - X_{first}}\right)^{\frac{X_{max} - X_{first}}{\lambda}} \exp\left(\frac{X_{max} - X}{\lambda}\right)$$

to compute the number of electrons N_e. Remember that X_{first} is the atmospheric depth where the first interaction takes place (equal to 80 g cm^{-2} in this case) and λ is the attenuation length (equal to 70 g cm^{-2}).

b) At the Auger Observatory the surface detectors are placed on a regular triangular grid, whose spacing is 1.5 km. By assuming that one needs at least 3 surface detectors, detecting more than 10 electrons/positrons to detect an air shower evaluate if the air shower from above can be detected with the Auger Observatory.
Hint: for the detectors consider a detecting surface A = 10 m^2.

c) With the same previous assumptions, would it have been possible for the KASCADE experiment to observe the same EAS?
Hint: KASCADE detectors were placed on a quadratic grid, with 13 m spacing, at sea level (i.e. $X = 1035$ g cm^{-2}). Consider for each detector a detecting surface $A = 4$ m^2.

Problem 44 Inverse Compton effect

X rays with a mean energy $E_\gamma = 5$ keV are observed from a region of interstellar matter. The x rays are produced through inverse Compton effect between electrons and photons of the 3 K microwave background.

What is the most probable energy of the 3 K background radiation in keV? Hint: use Wien's displacement law to estimate the most probable energy and note the discrepancy with the value provided at Problem 41.

Calculate the corresponding energy of the Compton electrons. Use the relation $E'_{ph} = \gamma_e^2 \cdot E_{ph}$. γ_e is the Lorentz factor of the electrons and E'_{ph} and E_{ph} the energies of the photons.

Problem 45 Gamma-ray detection on satellites

Gamma rays are detected with the GLAST/Fermi experiment. It comprises a tracker module with a thickness of 5 radiation lengths (Pb, $Z = 82$), followed by an electromagnetic calorimeter with a thickness of 8.6 radiation lengths (CsI, $\bar{Z} = 54$).

Estimate the energy of an incident photon (gamma ray) for which the maximum of the developing electromagnetic cascade is just inside the absorber.

Use a Heitler model to calculate the depth of the shower maximum

$$X^\gamma_{max} = X_0 \ln\left(\frac{E_0}{E^e_c}\right).$$

The critical energy of electrons in a solid material with nuclear charge number Z is given as (see also Problem 9)

$$E^e_c \approx \frac{610\ \text{MeV}}{Z + 1.24}.$$

To estimate the critical energy for a combination of materials, use the weighted average of the critical energy in each component.

The solutions will be discussed during the werkcollege on 08.12.2015 in HG02.749.
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Lecture web site: http://particle.astro.ru.nl/goto.html?astropart1516