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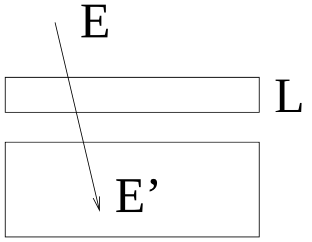
Werkcollege 2 – 21.09.2015

Problem 5 Silicon Detector

Cosmic-ray particles are measured with a silicon detector. The telescope consists of a thin silicon detector (thickness L) and a thick detector, in which the particles are absorbed, see sketch.



Particles with kinetic energy E loose the energy ΔE in the thin detector. The remaining energy E' is measured with the thick detector, i.e. $\Delta E = E - E'$. Nuclei with mass M and charge Z penetrate a distance R into the silicon detector, with

$$R_{Z,M}(E/M) = k\frac{M}{Z^2}\left(\frac{E}{M}\right)^{\alpha}.$$

It is $R_{Z,M}(E/M) - R_{Z,M}(E'/M) = L$.

Mass and charge of an atomic nucleus can be determined unambiguously with such a set-up. Show that

$$M = \left(\frac{k}{Z^2L}\right)^{1/(\alpha-1)} \left(E^{\alpha} - E'^{\alpha}\right)^{1/(\alpha-1)}$$

and

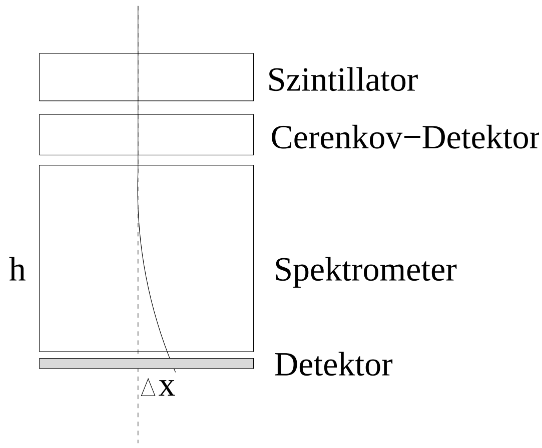
$$Z = \left(\frac{k}{L(2+\epsilon)^{\alpha-1}}\right)^{1/(\alpha+1)} \left(E^{\alpha} - E'^{\alpha}\right)^{1/(\alpha+1)}.$$

Assume $M/Z = 2 + \epsilon$. This is valid for light nuclei with ($Z < 30$).

see reverse side!

Problem 6 Magnet Spectrometer

Cosmic-ray particles are registered with a magnet spectrometer. It comprises a silicon detector to measure the charge Z of the particles, a Čerenkov detector to measure the particle velocity $\beta = v/c$, and a magnet spectrometer. The latter measures the rigidity of the particles. The rigidity is given as $R = pc/(Ze)$, with the particle momentum p, the speed of light c and the elementary charge e.



The homogeneous magnetic field ($B = 1$ T) inside the spectrometer has a hight of $h = 1$ m. The particle trajectory is measured with a space sensitive detector, the spatial resolution amounts to $\Delta x = 200$ μm.

The momenta of incoming particles are measured with the spectrometer. Calculate the maximum momentum p_{max}, which can be measured for the given spatial resolution for a proton (Z = 1, A = 1) and a helium nucleus (Z = 2, A = 4). Express the result in the unit [GeV/c]. It is 1 GeV/c = $5.34 \cdot 10^{-19}$ kg m/s.

Charge Z and mass M can be measured with such a detector. The momentum is given as $p = M\beta\gamma c$ with the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. Show that

$$M = \frac{RZe}{c^2} \cdot \sqrt{\frac{1}{\beta^2} - 1}.$$

Problem 7 Čerenkov detector

Charged particles travelling in a dielectric material at higher speed than the speed of light in that material emit radiation through the so-called "Čerenkov emission". The energy emitted dE through Čerenkov emission per unit of travelled length dx and unit of emitted frequency $d\nu$ is given by the Frank-Tamm formula

$$\frac{dE}{dx\ d\nu} = \frac{q^2}{2}\ \mu(\nu)\ \nu \left(1 - \frac{1}{\beta^2\ n^2(\nu)}\right)$$

where q is the particle charge, $n(\nu)$ is the refractive index, and $\mu(\nu)$ the electromagnetic permeability.

Estimate the kinetic energy threshold for a proton in air (n = 1.0003), water (n = 1.33), and plexiglas (n = 1.49). Evaluate the ratio of Čerenkov emission in plexiglas for a proton and an electron with kinetic energy E_{kin} = 400 MeV.

see reverse side!

Problem 8 Transition radiation detector

Transition radiation detectors (TRDs) are used in combination with other detectors to measure the energy and the mass of charged particles. Their working principle is based on the transition radiation effect, which takes place any time a charged particle passes through the boundary between two media or a discontinuity region inside a single medium. The emitted radiated energy E is proportional to the Lorentz factor γ ($E \propto \gamma$). The emitted X-ray photons can be detected if Čerenkov emission doesn't take place at such frequencies, otherwise they are overwhelmed by X-ray Čerenkov photons.

Evaluate the maximum refractive index in the X-ray range allowed for a TRD able to distinguish positrons and protons at energies up to 1 TeV.

X-ray photons are detected by a MWPC detector filled with gas through photoelectric effect. What is the best choice for that target gas? Please, motivate the answer.

Problem 9 Electromagnetic calorimeter

Electromagnetic calorimeters are used to measure the energy of photons and $e^\pm$ through multiple bremsstrahlung and pair-production interactions, which end into an electromagnetic shower composed of photons and $e^\pm$. The radiation length, i.e. the the average distance x that an electron needs to travel in a material to reduce its energy to $1/e$ of its original energy, for $e^\pm$ with energy above 1 GeV can be expressed by

$$X_0 = \frac{716\ A}{Z(Z+1)\ \ln\left(287/\sqrt{Z}\right)}\ \text{g cm}^{-2}$$

Evaluate the radiation length for a calorimeter made of tungsten and for one made of carbon.

The critical energy, defined as the energy at which the electron ionization losses and bremsstrahlung losses become equal, is given by

$$\epsilon = \frac{610}{Z+1.24}\ \text{MeV}.$$

The depth at which the electromagnetic shower reaches its maximum number of particles can be approximated as

$$X_{max} \simeq \ln\left(\frac{E}{\epsilon}\right) \cdot X_0.$$

The lateral extension of a shower is given by the Molière radius

$$R_M = 21\ \text{MeV}\frac{X_0}{\epsilon(\text{MeV})}\ \text{g cm}^{-2}.$$

Given an electromagnetic shower, started by a 1 TeV electron, evaluate the required linear thickness and width of a calorimeter made of tungsten with the following requirements: the shower should reach X_{max} and has to be laterally confined. Repeat the same computation for a calorimeter made of carbon.

The solutions will be discussed during the werkcollege on 21.09.2015 in HG03.082.
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Lecture web site: http://particle.astro.ru.nl/goto.html?astropart1516