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Problem 24 Hillas diagram

The Hillas condition describes the relation between the size and the magnetic field strength of a cosmic accelerator. It can be used to estimate the capability of astrophysical objects to accelerate particles to a certain energy.

In astrophysical units the Hillas condition is

$$B_{\mu G} \cdot L_{pc} > \frac{2 \cdot E_{15}}{Z \cdot \beta},$$

$B_{\mu G}$ is the strength of the magnetic field in μG, L_{pc} the size of the acceleration region in pc, E_{15} the energy of the particles in units of 10^{15} eV, Z the nuclear charge of the particles, and β the velocity of the particles in the center of mass reference system.

Draw a Hillas diagram, i.e. magnetic field strength vs. size for Pulsars, SNRs, Radio-Galaxies, and Sun spots.

The LHC accelerator at CERN is able to accelerate protons up to $E = 7$ TeV. By using the Hillas condition, compute the magnetic field which would be required to accelerate iron nuclei up to $E = 10^{20}$ eV. Is the found value feasible?

Problem 25 Pulsars emitting radiation as a lighthouse

Neutron stars quickly rotating and emitting a radio signal are called Pulsars. They are produced during SN type II explosions and are the remaining part of the collapsed core of the progenitor star (core external layers can be ejected during the chaotic collapsing phase). Their typical mass is $M_{Pulsar} \sim M_{Sun}$ and their typical radius is $R_{Pulsar} = 10$ km. By assuming a progenitor stellar core with $M = M_{Sun}$, radius $R = 12000$ km, and rotational period $T = 1$ d and assuming the core mass and angular momentum to be fully conserved, evaluate the rotational period of the resulting Pulsar.

The Pulsar is a rotating magnetic dipole, whose electromagnetic radiation power can be derived from the Larmor equation

$$-\frac{dE}{dt} = \frac{\mu_0 \; \Omega^4 \; p_{m_0}^2}{6\pi \; c^3},$$

where p_{m_0} is the component of the magnetic dipole perpendicular to the rotation axis, μ_0 the magnetic permeability of vacuum, and Ω the neutron star angular velocity.

By assuming an angle $\alpha = 45°$ between the magnetic and the rotation axis, $B =$

10^8 T, evaluate the emitted power through electromagnetic radiation by the pulsar. Hint: use

$$p_{m_0} = \frac{4\pi}{\mu_0} B R^3 \sin\alpha,$$

where $\mu_0 = 4\pi 10^7$ H m^{-1}.

Problem 26 Crab Nebula Pulsar

The Pulsar inside the Crab Nebula is one of the brightest objects on the sky. Its rotational energy loss has been evaluated to 6.4 10^{31} W. The pulsar rotation frequency has been measured to be 30.2 Hz and its mass $M = 1.5\ M_{Sun}$. By assuming $R = 10$ km and the magnetic field being perpendicular to the rotation axis, evaluate the magnetic field strength.

Neglecting any other possible interference to the Pulsar rotation (e.g. *starquakes*), evaluate the time needed for the Crab Nebula Pulsar to double its rotation period.

Problem 27 Cosmic Ray acceleration by Crab Nebula Pulsar

Particles can be accelerated by first order Fermi mechanism by a Pulsar until they stay within the region where the magnetic field lines of the neutron star are closed. The boundaries of this region are set by the rotational velocity of the field lines surrounding the Pulsar, which cannot exceed the speed of light c. Compute the diameter D of this region for the Crab Nebula Pulsar and the magnetic field intensity at this distance. What is the maximum energy of a proton confined in this region? What is the corresponding synchrotron radiation energy loss per time unit for this proton?

Hint: for synchrotron radiation use the standard formula

$$P(E, r) = \frac{e^2 c}{6\pi\epsilon_0 r^2} \left(\frac{E}{m_o c^2}\right)^4 \quad .$$

For an observer at large distance, the magnetic field is $B \propto r^{-2}$.

The solutions will be discussed during the werkcollege on 12.10.2015 in HG03.082.
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Lecture web site: http://particle.astro.ru.nl/goto.html?astropart1516