Radboud Universiteit Nijmegen
Afdeling Sterrenkunde
Jörg R. Hörandel
Antonio Bonardi

Astroparticle Physics – 2016/17

Werkcollege 1 – 07.09.2016

Problem 1 High-energy cosmic rays

1. The energy spectrum of high-energy cosmic rays is described by a power law of the form
$$\frac{dN}{dE} \propto E^{-\gamma} \begin{cases} \gamma = 2.7; & E < 10^{15} \text{ eV} \\ \gamma = 3.0; & E > 10^{15} \text{ eV} \end{cases}$$
Assume a flux of 4 particles/[m^2 s] at an energy of 10^{12} eV. Calculate the flux of particles at energies of 10^{15} eV [1/m^{2}y] and 10^{18} eV [1/km^{2}y].

2. The AMS experiment, which is installed onboard of the International Space Station, is the presently largest detector for the direct measurement of cosmic rays. It can detect protons with energy up to $\sim 10^{13}$ eV and its cost was 1.5 billion USD. By assuming a direct dependence between detector cost and covered area, estimate the cost of a future detector able to directly measure cosmic rays up to 10^{15} eV and compare this amount to the Global Gross Domestic Product (77 trillion USD in 2014).

3. In cosmic rays protons have been measured with energies of 10^{20} eV. Calculate the velocity of a tennis ball ($m = 80$ g) with the same kinetic energy.

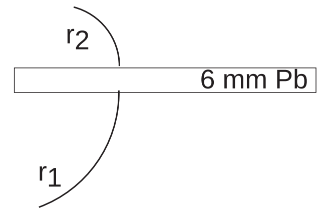
   The energy is carried by one proton in cosmic rays. Estimate the number of nucleons (i.e. protons and neutrons) in the tennis ball. Hint: suppose neutron mass equal to the proton one.

Problem 2 Energy loss according to Bethe-Bloch
Protons with a kinetic energy of 50 MeV impinge on a layer of aluminum and lead, respectively.

1. Considering the thickness of the crossed materials equal to 1 g/cm^2, In which material loose the particles more energy?
Hint: Use in the Bethe-Bloch formula $\langle E_B \rangle \approx 10 \cdot Z_2$ eV as average binding energy of the target electrons. The Bethe-Bloch formula is given as
$$\frac{dE}{dx} = -\frac{Z_1^2 e^4 n_e}{4\pi \cdot \epsilon_0^2 v^2 m_e} \cdot \left[\ln \frac{2 m_e v^2}{\langle E_B \rangle} - \ln(1-\beta^2) - \beta^2 \right]$$
where Z_1 is the impinging particle electric charge, e the electron charge, m_e the electron mass, n_e the electron density of the crossed material, and ϵ_0 the vacuum permittivity.

2. How does the result change if we assume the same geometrical thickness of the target (i.e. same thickness in cm)? The densities are $\varrho_{\text{Al}} = 2.70 \text{ g/cm}^3$, $\varrho_{\text{Pb}} = 11.34 \text{ g/cm}^3$).

Problem 3 Discovery of the positron



Through observations of particle tracks in a cloud chamber positrons have been discovered in 1933 by Carl D. Anderson. The set-up was in a magnetic field with a strength of $B = 15000$ G. Two particles were observed. The radii of the trajectories were $r_1 = 13.3$ cm and $r_2 = 5.2$ cm.
Calculate the energy of both particles expressed in MeV, which is the typical unit of particle physics. Hint: assume that particles are singly charged, i.e. carry one elementary charge.

Problem 4 Energy loss of electrons in lead

The energy loss of electrons in lead is described by

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_{ion} + \left(\frac{dE}{dx}\right)_{rad}.$$

I.e. with a contribution of ionization losses and radiation losses. Sketch qualitatively the dependence of the energy losses through ionization and radiation losses as function of the Lorentz factor $\gamma = E/m_e\, c^2$.

What is the meaning of the critical energy E_{krit}?

The specific energy losses for minimum ionizing particles is $dE/dx_{min} = 1.123$ MeV/[g cm^2]. The critical energy for electrons in lead is $E_{krit} = 6.9$ MeV. The residual energy (after radiation losses) for $E_e >> E_{krit}$ is described by

$$E_e(x) = E_e(0) \exp\left(-\frac{\rho\, x}{l_{rad}}\right).$$

The parameters for lead are $\rho_{Pb} = 11.35$ g/cm^2 and $l_{rad} = 58$ kg/m^2.

Calculate the energy loss through ionization and radiation losses of an electron with an energy of 60 MeV in 6 mm lead. Compare the values with the energy loss in problem 3.

The solutions will be discussed during the werkcollege on 07.09.2016 in HG 01.057.
Student assistant: Antonio Bonardi a.bonardi@astro.ru.nl
Lecture web site: http://particle.astro.ru.nl/goto.html?astropart1617