

galactic chemical evolution (GCE)

to model chemical evolution of galaxies,
one needs

- i) initial conditions
- ii) end products of stellar evolution
- iii) birthrate of stars (initial mass function)
- iv) model of total star formation rate
- v) mixing of ISM & IGM

1. Initial conditions

mostly pure gas with primordial abundances
from BB

2. End products of stellar evolution

Contributions of different stars depend on
their initial mass,
chemical composition,
mass loss history,
effect of close binaries (SN Ia)

3. Initial mass function IMF

describes relative birth rates of stars
with different initial mass in M_{\odot}
at a given time and a given region
such as the solar cylinder

Table 7.2. Theoretical yields of selected species from massive stars

Species	$20 M_{\odot}$			$25 M_{\odot}$			Yield	
	Ref.	1	2	3	1	2	3	
^{12}C			(0.29)	(0.13)		(0.53)	(0.22)	(4.1E-4)
2.4E-3	0.22	0.32	0.21	0.41	0.61	0.14	6.8E-4	
^{14}N			(3.7E-3)	(0.013)		(4.6E-3)	(9.2E-3)	(2.8E-5)
7.8E-4	0.064	0.062	0.072	0.081	0.072	0.084	2.4E-4	
^{16}O			(1.4)	(2.2)		(2.3)	(3.8)	(8.0E-3)
6.4E-3	2.2	1.4	1.0	3.3	2.4	2.2	6.1E-3	
^{20}Ne			(0.54)	(0.63)		(0.53)	(1.2)	(1.4E-3)
1.2E-3	0.07	0.54	0.29	0.54	0.72	0.65	1.7E-3	
^{22}Ne			(0.0020)	(0.0012)		(0.0028)	(0.0015)	(3.4E-6)
8.5E-5	0.020	0.030	0.009	0.028	0.051	0.017	2.8E-5	
^{23}Na			(3.2E-3)	(1.8E-3)		(3.6E-3)	(8.1E-3)	(8.2E-6)
3.8E-5	2.2E-3	0.013	0.012	0.013	0.024	0.014	5.4E-5	
^{24}Mg			(0.16)	(0.24)		(0.15)	(0.18)	(6.7E-4)
5.5E-4	0.067	0.13	0.069	0.12	0.13	0.21	4.1E-4	
^{27}Al			(4.4E-3)	(7.0E-3)		(3.7E-3)	(5.0E-3)	(2.5E-5)
6.6E-5	0.012	0.019	8.6E-3	0.026	0.031	0.023	6.5E-5	
^{28}Si			(0.10)	(0.13)		(0.24)	(0.12)	(7.1E-4)
7.3E-4	0.44	0.16	0.094	0.35	0.24	0.12	4.6E-4	
^{32}S			(0.044)	(0.056)		(0.11)	(0.055)	(2.9E-4)
3.9E-4	0.20	0.066	0.048	0.15	0.12	0.044	2.0E-4	
^{36}Ar			(0.009)	(0.010)		(0.021)	(0.009)	(4.2E-5)
8.8E-5	0.05	0.011	0.007	0.023	0.021	0.007	3.0E-5	
^{39}K			(1.9E-5)	(1.4E-5)		(3.8E-5)	(1.5E-5)	(1.4E-7)
4.1E-6	1.0E-4	1.4E-4	1.4E-4	5.8E-4	3.7E-4	6.8E-5	3.4E-7	
^{40}Ca			(0.008)	(0.009)		(0.019)	(0.008)	(3.3E-5)
7.0E-5	0.027	0.009	0.0050	0.017	0.016	0.0057	2.4E-5	
^{48}Ti			(1.8E-4)	(1.4E-4)		(2.0E-4)	(1.2E-4)	(5.0E-7)
2.5E-6	2.3E-4	1.8E-4	7.4E-5	2.0E-4	2.0E-4	1.7E-4	5.7E-7	

1. Rauscher *et al.* (2002); $Z = 0.02$, including wind mass loss.

2. Chieffi and Limongi (2004); $Z = 0.02$. Numbers in brackets are for $Z = 0.001$. No wind mass loss assumed.

3. Nomoto *et al.* (2006), including wind mass loss, $Z = 0.02$, with numbers in brackets for $Z = 0.001$.

Species are given with their proto-solar abundance by mass fraction, after Lodders (2003). The last column gives the yield calculated by Nomoto *et al.* for core-collapse supernovae within a Salpeter IMF between mass limits of 0.07 and $50 M_{\odot}$.

Table 7.3. Primary element production from massive stars with modest mass loss

M_{init}	M_{fin}^a	M_{α}^b	M_{CO}^c	He	C	O	Z
120	81	81	59	9.8	0.88	35	42
85	62	62	38	8.1	0.72	23	27
60	47	28	25	6.0	0.70	14	17
40	38	17	14	4.2	0.55	6.8	10
25	25	9	7	3.5	0.40	2.4	4.4
20	19	7	5	2.1	0.30	1.3	2.9
15	15	5	3	1.6	0.20	0.46	1.5
12	12	4	2	1.4	0.10	0.15	0.8
9	9	3	2	1.0	0.06	0.004	0.3
5	5	1	1	0.45			
3	3			0.09			

^a Final mass at end of carbon-burning (or helium-burning for lower masses).

^b Mass of He core at end of carbon- or helium-burning.

^c Mass of CO core at end of carbon- or helium-burning.

Source: Maeder (1992) for the case $Z = 0.001$, $Y = 0.24$.

Table 7.4. Primary element production from stars with drastic mass loss

M_{init}	M_{fin}	He		C		O		Z	
		<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>
120	2.4	42.7	-0.1	8.0	0.3	0.0	0.2	10.1	0.7
85	3.5	16.7	-0.4	13.5	0.4	4.0	0.6	19.3	1.6
60	3.0	13.5	-0.3	7.2	0.3	1.4	0.4	9.8	1.2
40	3.6	6.1	-0.4	4.9	0.4	2.1	0.6	8.0	1.6
25	11.3	1.5	0.6	0.30	0.32		2.6		4.5
20	14.0	1.6	1.5		0.22		1.3		2.7
15	13.6	1.4	1.3		0.14		0.4		1.3
12	11.5		1.2		0.07		0.1		0.7
9	8.6		0.9		0.03		0.0		0.2
5	4.9		0.40						
3	3.0		0.07						

^a Sum of amounts freshly produced and expelled in wind and in final ejecta (SN or PN).

^b Amount freshly produced and expelled in final ejecta (SN or PN).

Source: Maeder (1992) for the case $Z = 0.02$, $Y = 0.28$, high mass-loss rates.

Table 7.5. *He, C and N production from intermediate-mass stars: Version 1*

M_{init}	Z	M_{rem}	${}^4\text{He} \times 100$	${}^{12}\text{C} \times 1000$	${}^{13}\text{C} \times 10^4$	${}^{14}\text{N} \times 1000$	${}^{16}\text{O} \times 10^4$
			3	2.4	0.27	0.8	64
8	0.004	1.20	3.3	0.33	1.3	9	-3.2
	0.008	1.20	4.1	-0.05	1.6	11	-10
	0.02	1.15	3.9	-0.92	1.8	11	-16
	0.001	1.15	5.6	1.7	2.1	18	9.6
	0.004	0.98	2.5	0.3	0.88	6.4	2.6
	0.008	0.97	2.6	0.0	0.97	6.4	0.7
5	0.02	0.92	3.1	-0.7	1.4	7.3	-2.9
	0.001	1.03	4.8	2.5	4.7	14	13
	0.004	0.91	2.0	0.8	2.7	4.5	2.8
	0.008	0.90	2.6	0.7	3.0	4.4	0.6
	0.02	0.79	3.1	4.6	0.70	1.8	-2.2
	0.001	0.89	3.6	14	-0.13	0.07	12
4	0.004	0.66	3.2	7.4	0.13	0.44	5.1
	0.008	0.64	3.4	6.8	0.24	0.78	3.0
	0.02	0.62	3.3	5.1	0.67	1.7	-0.01
	0.001	0.71	4.4	13	-0.22	0.07	12
	0.004	0.62	2.7	4.9	0.11	0.34	3.6
	0.008	0.61	2.5	4.4	0.22	0.58	2.5
3	0.02	0.60	2.0	2.4	0.60	1.3	0.78
	0.001	0.64	3.5	9.5	-0.19	0.037	8.4
	0.004	0.60	1.9	2.2	0.10	0.20	2.1
	0.008	0.59	1.9	2.5	0.18	0.36	2.2
	0.02	0.59	1.6	0.80	0.51	0.99	3.7
	0.001	0.59	1.8	1.7	-0.14	0.028	1.5
2	0.004	0.58	1.5	0.68	0.074	0.12	0.64
	0.008	0.58	1.3	-0.17	0.14	0.24	0.54
	0.02	0.58	1.9	-0.12	0.48	0.77	9.8
	0.001	0.58	1.3	-0.021	-0.13	0.025	-0.02
	0.004	0.58	1.2	-0.09	0.065	0.10	-0.01
	0.008	0.58	1.2	-0.15	0.12	0.20	0.28
1.5	0.02	0.57	1.0	-0.42	0.31	0.48	-0.39
	0.001		1.4	1.5	0.15	1.4	2.0
	0.004		1.5	1.6	0.36	1.4	1.5
	0.008		1.6	1.5	0.45	1.6	0.5
	0.02		1.7	1.0	0.52	1.8	1.0
	Over-all yield						

After van den Hoek & Groenevegen (1997), with their standard parameters, except that for $Z = 0.001$ the mass loss parameter η in the Reimers (1975) formula is 1 instead of 4. The lowest block represents the overall yields contributed by intermediate-mass stars in a population governed by a Salpeter mass function between 0.1 and $120 M_{\odot}$, calculated from the above stellar yields by Mariño (2001), to be compared with solar abundances (by mass) given in the second line.

Table 7.6. *He, C and N production from intermediate-mass stars: Version 2*

M_{init}	Z	M_{rem}	${}^4\text{He} \times 100$	${}^{12}\text{C} \times 1000$	${}^{13}\text{C} \times 10^4$	${}^{14}\text{N} \times 1000$	${}^{16}\text{O} \times 10^4$
5	0.004	1.04	17	1(4)	6.9(6.5)	47(43)	1(31)
	0.008	1.03	32	-3(2)	3.2(2.7)	22(14)	-27(13)
	0.019	1.00	29	4(11)	37(23)	13(3)	-40(14)
	0.004	1.00	20	2(4)	7(7)	44(42)	23(35)
4	0.008	0.98	14	8(10)	34(29)	14(11)	5(20)
	0.019	0.90	13	16(20)	2(2)	5(0)	-12(18)
	0.004	0.97	22	78	0.58	0.62	62
3	0.008	0.89	16	47	0.54	1.46	29
	0.019	0.79	14	36	1.38	3.36	5.8
	0.004	0.75	18.5	71	0.11	0.23	58
2	0.008	0.68	9.0	29	0.32	0.67	19.6
	0.019	0.63	6.0	16	0.88	1.53	5.3
	0.004	0.64	8.1	30	0.095	0.083	25
1.5	0.008	0.61	3.3	8.4	0.25	0.18	6.4
	0.019	0.58	1.7	2.4	0.58	0.56	1.5
	0.004	0.61	0.59	-0.020	0.046	0.018	0.0011
1.0	0.008	0.61	3.3	8.4	0.25	0.32	6.4
	0.019	0.54	0.70	-0.078	0.21	0.069	0.00
	0.004	0.59	0.048	0.0031	-0.096	0.0080	0.00
0.9	0.008	0.58	0.49	-0.021	-0.058	0.018	-0.002
	0.019	0.54	0.51	-0.045	0.13	0.038	0.00
	Over-all yield		3.1	9	1.0	1.0	7.5
all	0.008		1.8	4.0	0.40	0.40	2.5
	0.019		1.4	2.5	0.32	0.40	0.1

After Marigo (2001) with standard parameters. Numbers in brackets denote the primary contribution from masses 4 and $5 M_{\odot}$. (Primary contributions are necessarily positive, whereas secondary ones can be either positive or negative.) Overall yields defined as in Table 7.5.

Table 7.7. *Element production from SN Ia*

Species	Mass/ M_{\odot}	$[X_i/X_{56}]^a$
^{24}Mg	0.09	-1.1
^{28}Si	0.16	-0.3
^{32}S	0.08	-0.4
^{36}Ar	0.02	-0.3
^{40}Ca	0.04	0.1
^{54}Fe	0.14	0.6
^{56}Fe	0.61	0.0
^{58}Ni	0.06	0.4
Cr-Ni	0.86	

^a Logarithmic element: ^{56}Fe ratio relative to solar.
Source: Nomoto *et al.* (1984), model W7; see
also Thielemann *et al.* (1986).

present day mass function

$$PDMF(\log m) = \frac{dN}{dM_v} \cdot Z_L(M_v) \cdot \frac{dM_v}{d\log m}$$

$\frac{dN}{dM_v}$ luminosity function for local volume

$\frac{dM_v}{d\log m}$ mass-luminosity relation

$h(M_v)$ scale height

SIMF depends on lifetime of star

$m \leq 0, 9$ long-lived

$$\text{SIMF} = PDMF / (\tau \langle \Psi(t) \rangle)$$

τ age of galactic disc

$\langle \psi(t) \rangle$ mean rate of star formation
over that time

$m > 2$ short-lived

$$\text{IMF} = \text{PDMF} / \left\{ \bar{\tau}_{\text{ms}}(m) \langle \psi(T) \rangle \right\}$$

no change in $\psi(t)$ on short
time

in intermediate masses

$$\text{IMF} = \text{PDMF} / \int_{\bar{\tau}_{\text{ms}}(m)}^T \psi(t) dt$$

IMFs typical $m \phi(m) \propto m^{-\alpha}$

e.g. $m \phi(m) = 0,17 m^{-1,35}$

$120 M_\odot$

$$\int_{0,1 M_\odot}^{120 M_\odot} m \phi(m) dm = 1 \quad \text{normalization}$$

4. Star formation rate SFR

typically simple laws are used

e.g. $\Psi = \Psi_0 e^{-\nu t} \quad \frac{1}{\nu} = 10^8 a \text{ to } 2 \cdot 10^9 a$

estimate ν :

all energy radiated corresponds to
synthesis of nuclei from hydrogen

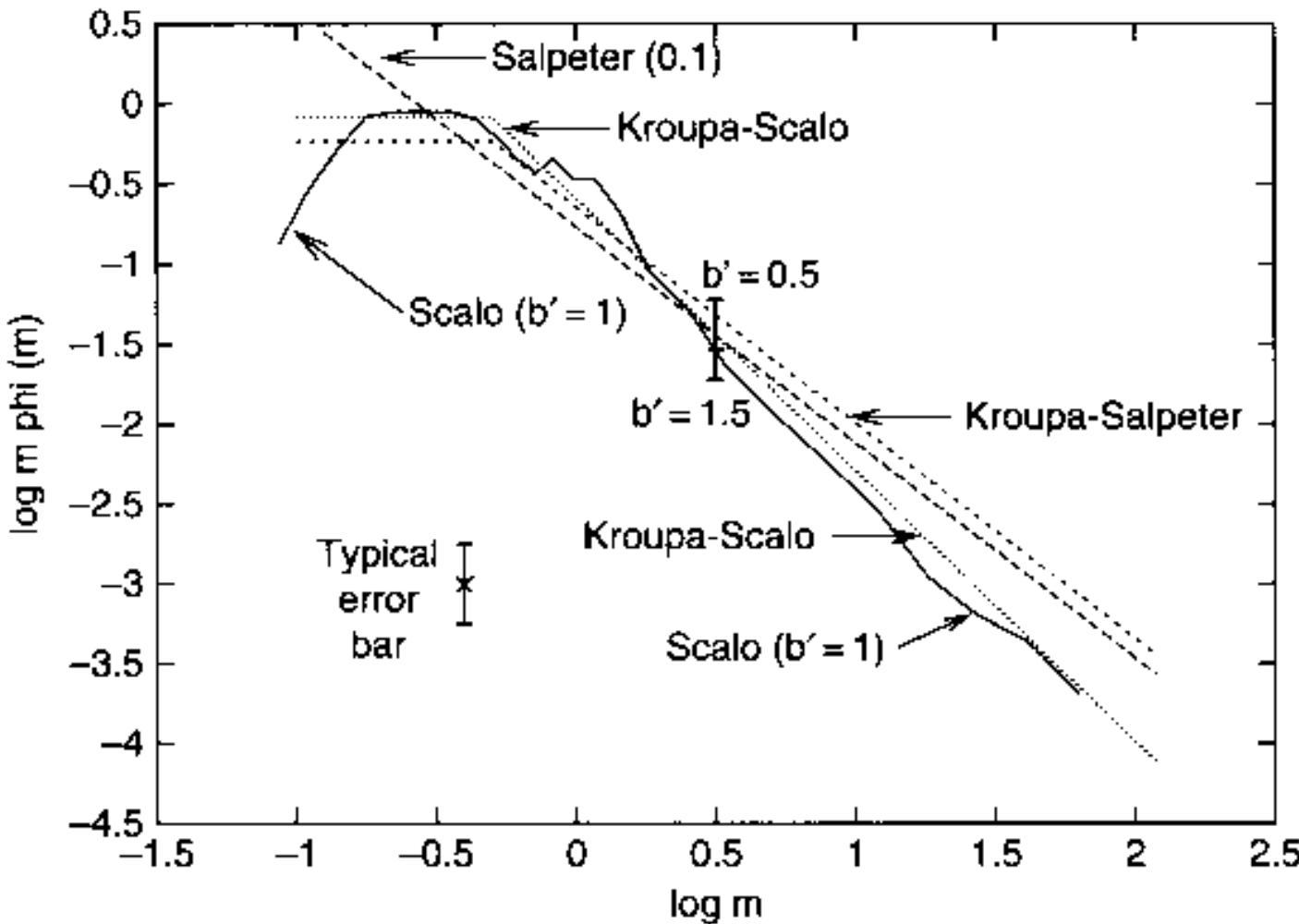


Fig. 7.1. Local IMFs after Scalo (1986) with $b' = 1$, Salpeter (1955) extended down to $0.1 M_{\odot}$ and versions with a flat slope below $0.5 M_{\odot}$, consistent with suggestions by Kroupa (2002). The IMFs are normalized to a total mass of $1 M_{\odot}$, between adopted lower limits (zero in the case of the flattened functions) and $120 M_{\odot}$.

$$L \approx -0.007 M_c^2 \frac{dx}{dt}$$

→ overall present-day abundance in whole system

$$(1) \quad \langle z + \Delta Y \rangle \approx 0.005 \frac{L_1}{M} \frac{e^{\sqrt{T}-1}}{\sqrt{2}}$$

L_1 present-day bolometric luminosity

in solar neighbourhood

10% of luminous matter for white dwarfs

$$z \approx 0.01$$

$$\frac{dY}{dz} \approx 2.5$$

$$(2) \quad \Rightarrow \text{estimate} \quad \langle z + \Delta Y \rangle \approx 0.15$$

$$\frac{L_1}{M} \sim 1 \text{ in cgs units}$$

(\propto) $L(z) \Rightarrow$ exponential decay in
bolometric luminosity

$$z^{-1} \sim 7 \cdot 10^9 a$$

5. galactic context

main stellar population groups:

- disk
- bulge
- halo
- solar cylinder

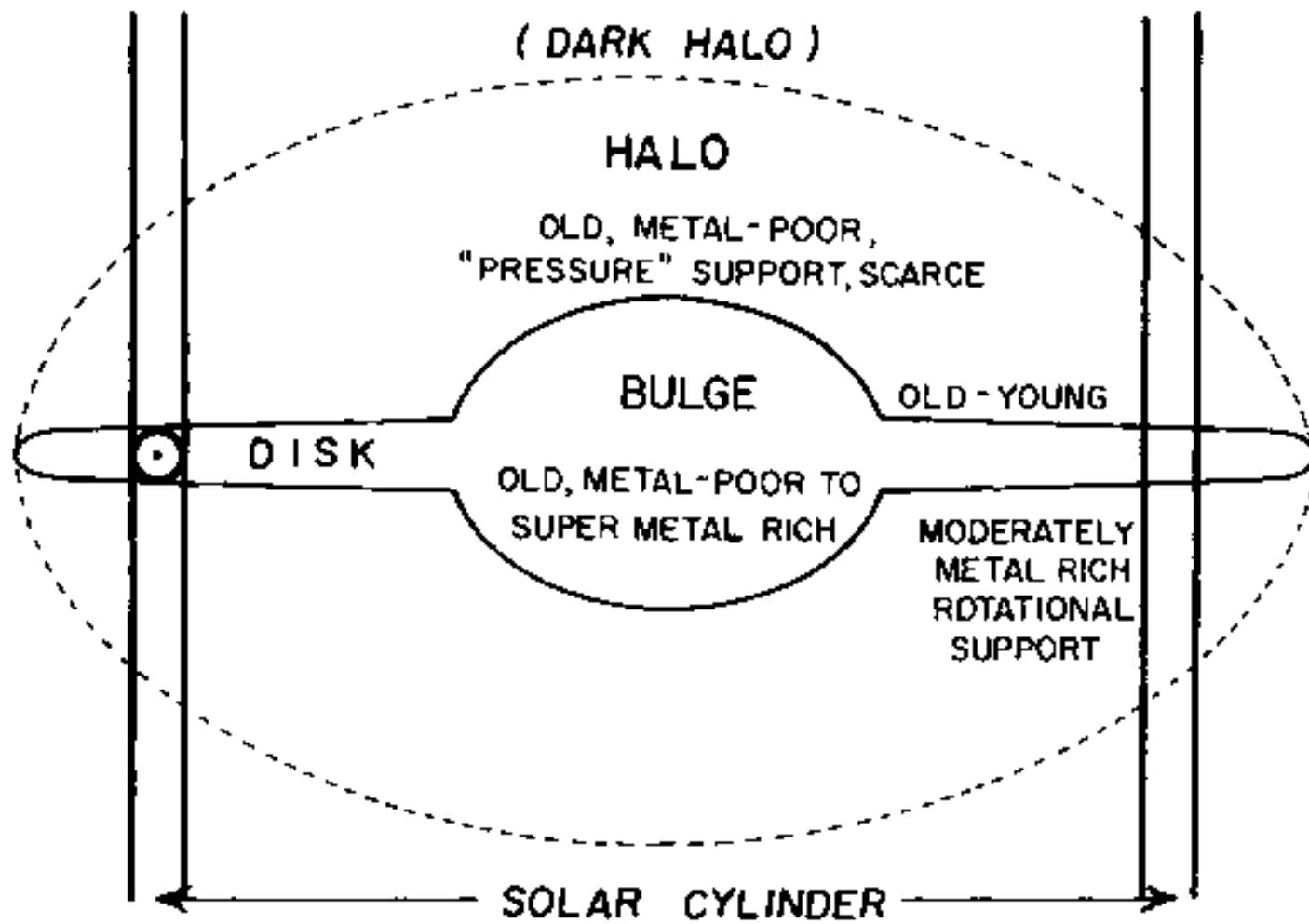


Fig. 3.38. Schematic cross-section through the Galaxy.

Table 7.9. Some properties of the Galaxy and the solar cylinder

	Galaxy	Solar cylinder
Age		10 to 15 Gyr
Mass now in stars ^a	$7 \times 10^{10} M_{\odot}$	$45 M_{\odot} \text{ pc}^{-2}$
Mass now in gas ^b	$\sim 7 \times 10^9 M_{\odot}$	$7 \text{ to } 14 M_{\odot} \text{ pc}^{-2}$
Gas fraction	~ 0.1	0.14 to 0.25
Surface brightness $(M/L_V)/(M/L_V)_{\odot}^a$	5	$23m_V, m_{bol} \text{ arcsec}^{-2}$ 3
Processes tending to deplete the gas:		
Average past SFR	$(5 \text{ to } 7)\alpha^{-1} M_{\odot} \text{ yr}^{-1}$	$(3 \text{ to } 4.5)\alpha^{-1} M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
Gas consumption time	$\sim 1 \text{ Gyr}$	1.5 to 5 Gyr
Processes tending to restore the gas:		
Mass ej. from AGB+PN ^c		$0.8 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
Mass ej. from O stars ^d		$\sim 0.05 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
Mass ej. from SN ^d	$\sim 0.15 M_{\odot} \text{ yr}^{-1}$	$\sim 0.05 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$
(Total mass ejection from stars)		$\sim 1 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$)
Net inflow from IGM ^e	$\leq 2 M_{\odot} \text{ yr}^{-1}$	$\leq 1 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$

Sources: ^a Binney & Tremaine (1987); ^b Kulkarni & Heiles (1987); ^c Jura (1989);

^d Pottasch (1984); ^e Lacey & Fall (1985).

GCE equations

total mass M (without non-baryonic DM)

mass of gas g

mass in form of stars s

abundance of elements z

system mass $M = g + s$

$$\text{and } \frac{dM}{dt} = F - E$$

F rate of accretion
from outside

E rate of ejection
e.g. gal. wind

mass of gas

$$\frac{dg}{dt} = F - \bar{E} + e - \Psi$$

Ψ star formation rate

e ejection rate of

matter from stars

mass of stars

$$\frac{ds}{dt} = \Psi - e$$

abundance of stable element in gas

governed by

$$\frac{d}{dt} (g\tau) = e_\tau - \tau_\Psi + \tau_F \cdot F - \tau_E \cdot E$$

↑ ↑ ↑ ↑
 loss due to gal. wind
 inflow of material
 loss to ISM by star formation
 total amount ejected from stars

recycling approximation

one assumes that all processes involving stellar evolution, nucleosynthesis & recycling take place instantaneously

mass of all stars that have been born
up to t

$$S(t) = \int_0^t \Psi(t') dt'$$

mass still in form of stars (or compact remnants)

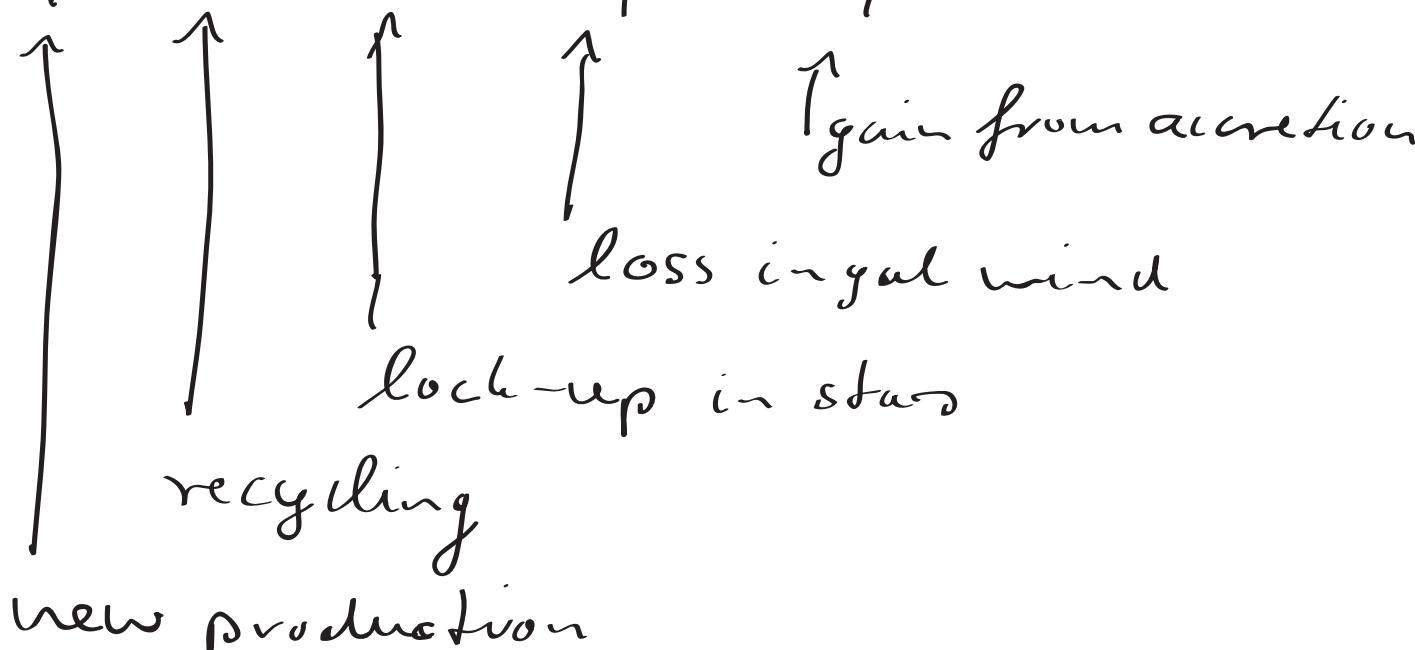
$$s(t) = \alpha S(t)$$

$$M(t) = s(t) + g(t) = M_0 - M_{ej} + M_{acc}$$

and $\frac{dg}{dt} = \bar{F} - E - \frac{ds}{dt}$

or $\frac{dg}{ds} = \frac{\bar{F} - E}{\alpha \Psi}$

assuming a homogeneous ISM, the abundance in the gas of a stable element is governed by

$$\frac{d}{ds} (g\tau) = q + R\tau - \tau - \tau_E \frac{E}{\zeta} + \tau_F \frac{F}{\zeta}$$


The diagram illustrates the components of the mass balance equation. It features five vertical arrows pointing towards the right side of the equation:

- A downward arrow under the term q , labeled "new production".
- An upward arrow under the term $R\tau$, labeled "recycling".
- A downward arrow under the term τ , labeled "loss in gal wind".
- A downward arrow under the term $\tau_E \frac{E}{\zeta}$, labeled "lock-up in stars".
- An upward arrow under the term $\tau_F \frac{F}{\zeta}$, labeled "gain from accretion".

Mixing processes in the interstellar medium

how well mixed is the ISM?

metals are ejected from SN in form of hot gas

$$r = 50 \text{ pc} \quad t \sim 10^5 \text{ a}$$

$$\rho = 1 \text{ atom/cm}^3 \rightarrow \text{mass in sphere} \sim 10^4 M_\odot$$

single SN expelling $2 M_\odot$ of oxygen

would contribute a mass fraction

$$\delta z_0 \sim 2 \cdot 10^{-4} \text{ or } z_0/30$$

\rightarrow it takes ~ 30 events to build up present abundance

Poisson \sqrt{N} fluctuations: $\pm 20\%$ variations

gas smoothed by

turbulence from differential rotation

molecular diffusion