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Particles and the Cosmos – 2020/21

Werkcollege 1 – Interactions with matter

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Comments and remarks on Problem 2

Problem 2 Energy loss according to the Bethe-Bloch formula

Assume two protons with kinetic energy of 60 MeV impinging on a layer of carbon and iron, respectively.

a) We assume the thiackness of the material being 1 g/cm^2. In which material loose the particles more energy?
Hint: for carbon consider $Z = 6$ and $A = 12$, for iron consider $Z = 26$ and $A = 56$. Use the Bethe-Bloch formula for dense media, as given in the lecture (and the book from Stanev)

$$\frac{dE}{dx} = -L\frac{Z^2}{\beta^2}(B + 0.69 + 2\ln\gamma\beta + \ln W - 2\beta^2 - \delta).$$

See lecture notes for the definition of the terms and constants.

b) How does the result change if we assume a geometrical thickness 1 cm for both targets?
Hint: the densities of the materials are given in the tables at the end of the lecture.

c) From the given Bethe-Bloch formula, compute the kinetic energy E_{kin} of a proton traveling through an iron absorber ($Z = 26$, $A = 56$) at the minimum ionization energy.
Hint: use $E_{kin} = (\gamma - 1) \cdot m_0c^2$ with $m_0c^2 = 938$ MeV and use a numeric approximation to solve the exercise.

I appologize, there are several issues with this Problem and the corresponding slides during the lecture.

- There seems to be a typo in the book of Stanev. The equation should read as

$$\frac{dE}{dx} = -L\frac{z^2}{\beta^2}(B + 0.69 + 2\ln\gamma\beta + \ln W - 2\beta^2 - \delta).$$

The charge z^2 should be the charge (squared) of the impinging particle (the projectile) and not the nuclear charge number of the absorber material Z. So small z insetad of Z.

- This approximation is only valid for large particle momenta (large Lorentz factors γ). In our example with a proton of 60 MeV we have $\beta = 0.34$ and $\gamma = 1.06$, so the approximation DOES NOT apply.
- The values for L are a factor 1000 too big on the lecture slides. The table should read

Element	I, eV	L	B	C
Hydrogen	21.8	0.152	21.07	-9.50
Helium	44.0	0.077	19.39	-2.13
Carbon	77.8	0.077	18.25	-3.22
Nitrogen	90.9	0.077	17.94	-10.68
Oxygen	104	0.077	17.67	-10.80
Iron	286	0.072	15.32	-4.62

There are (at least) two ways out of this: a) we use a proton of 60 GeV for our calculations and use the approximation (with z^2) or b) we use the full dE/dx equation given in the lecture.

a) Solution for a proton with (kinetic) energy of 60 GeV

The Lorentz factor and kinetic energy are correlated by the following formula

$$E = m_0 \, \gamma \, c^2 = E_{kin} + m_0 \, c^2$$

Then, for a proton (mass 938.5 MeV) with $E_{kin} = 60$ GeV,

$$\gamma = \frac{60000 + 938.5}{938.5} = 64.93$$

and

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9999$$

We use the approximation

$$\frac{dE}{dx} = -L\frac{z^2}{\beta^2}\left(B + 0.69 + 2\ln\gamma\beta + \ln W - 2\beta^2 - \delta\right).$$

Following the book of Stanev, we use

$$L = 0.0765 \left(\frac{2Z}{A}\right) \frac{\text{MeV}}{\text{g/cm}^2}.$$

For carbon with ($Z = 6$ and $A = 12$) we obtain $L = 0.077$ MeV/g/cm^2. $B = 18.25$ and $\delta = 2\ln\gamma\beta + C$, with $C = -3.22$. The two terms $2\ln\gamma\beta$ cancel each other. We assume a proton with $E = 60$ GeV, thus, we get $W \approx E/2 \approx 30$ GeV. So we have between brackets:

$$\left(\underbrace{18.25}_{B} + 0.69 + \underbrace{\ln 30000}_{10.31} + \underbrace{3.22}_{C}\right) = 32.47.$$

So we get

$$\frac{dE}{dx} = -0.077 \cdot \frac{1}{1} \cdot 32.47 \frac{\text{MeV}}{\text{g/cm}^2} = 2.5 \frac{\text{MeV}}{\text{g/cm}^2}.$$

This is a reasonable value for the energy loss in the region of the relativistic raise.

b) Solution for a proton with (kinetic) energy of 60 MeV and the full equation

The Lorentz factor and kinetic energy are correlated by the following formula

$$E = m_0 \gamma c^2 = E_{kin} + m_0 c^2$$

Then, for a proton (mass 938.5 MeV) with $E_{kin} = 60$ GeV,

$$\gamma = \frac{60 + 938.5}{938.5} = 1.06$$

and

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.34 \quad \text{thus} \quad v = c\beta = 1.02^8 \frac{\text{m}}{\text{s}}.$$

For these values, we canNOT use the approximation. Thus, we will use the full equation as given in the lecture and the book of Stanev (2.5).

$$\frac{dE}{dx} = -\frac{N_A Z}{A} \frac{2\pi (ze^2)^2}{Mv^2} \left[\ln \frac{2Mv^2\gamma^2 W}{I^2} - 2\beta^2 \right].$$

I do not see how we get the correct units, MeV/(g/cm^2) from the first term (before the [. . .]). Therefore, I propose to use a slightly modified version of the Bethe-Bloch equation as given in the original solutions to this problem.

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Lecture web site: http://particle.astro.ru.nl/goto.html?partcos2021