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Particles and the Cosmos – 2020/21
Werkcollege 1 – Interactions with matter
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Problem 1 Maximum energy transfer to electrons during the ionization process

When a charged particle impinges an absorber material, a part of its kinetic energy is transfered to the electrons of the absorber material. Since this energy transfer is usually larger than the electrons binding energy, the target atoms are ionized, and hence this process is called “ionization”.

a) Verify that the maximum energy transfer from the traveling particle to an electron of the absorber material is equal to

$$E_{kin}^{max} = \frac{2m_e p^2}{m_0^2 + m_e^2 + 2m_e \frac{E}{c^2}}$$

where m_e is the electron mass, m_0 the impinging particle mass at rest, p the impinging particle momentum, E the imping particle total energy, and c the speed of light in vacuum.

The maximum energy transfer E_{kin}^{max} is named W in the Bethe-Bloch formula used in the lecture (and in the nomenclature of Stanev).

b) Compute E_{kin}^{max} for a proton and a muon with a kinetic energy of 10 GeV.
Hint: $m_0^e = 511$ keV/c^2, $m_0^p = 938$ MeV/c^2, and $m_0^\mu = 106$ MeV/c^2.

Problem 2 Energy loss according to the Bethe-Bloch formula

Assume two protons with kinetic energy of 60 MeV impinging on a layer of carbon and iron, respectively.

a) We assume the thiackness of the material being 1 g/cm^2. In which material loose the particles more energy?
Hint: for carbon consider $Z = 6$ and $A = 12$, for iron consider $Z = 26$ and $A = 56$. Use the Bethe-Bloch formula for dense media, as given in the lecture (and the book from Stanev)

$$\frac{dE}{dx} = -L\frac{Z^2}{\beta^2}(B + 0.69 + 2\ln\gamma\beta + \ln W - 2\beta^2 - \delta).$$

See lecture notes for the definition of the terms and constants.

b) How does the result change if we assume a geometrical thickness 1 cm for both targets?
Hint: the densities of the materials are given in the tables at the end of the lecture.

c) From the given Bethe-Bloch formula, compute the kinetic energy E_{kin} of a proton traveling through an iron absorber ($Z = 26$, $A = 56$) at the minimum ionization energy.
Hint: use $E_{kin} = (\gamma - 1) \cdot m_0 c^2$ with $m_0 c^2 = 938$ MeV and use a numeric approximation to solve the exercise.

Problem 3 Ionization in a thin absorber

Given the stochastic nature of the ionization process, large fluctuations occur in the distribution of the transferred energy to the absorber electrons. This becomes evident in case of thin absorber, like a sub-millimeter thick iron slab or gases. In those cases the energy loss of the impinging particle is well described by the Landau formula

$$L(\lambda) = \frac{1}{\sqrt{2\,\pi}} \cdot \exp\left[-\frac{1}{2}\left(\lambda + e^{-\lambda}\right)\right]$$

where λ is defined as

$$\lambda = \frac{\Delta E - \Delta E^W}{\xi}$$

where ΔE is the actual energy loss, ΔE^W is the most probable value of energy loss and ξ is given by

$$\xi = 2\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}\rho l \propto \rho\ l$$

where ρ is the absorber density and l is the absorber geometrical thickness.

a) Compute the parameter ξ for a relativistic single-charged particle (i.e. $\beta = 1$) in a layer of iron 0.1 mm thick.
Hint: for iron consider $A = 56$, $Z = 26$, $\rho = 7.9\ 10^{-3}$ g cm^{-3}.

b) In case of an electron with kinetic energy equal to 2.5 MeV in a 0.1-mm layer of iron the most probable energy loss can be written as

$$E^W = \xi\left[\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I^2}\xi\right) - \beta^2\right]$$

Compute E^W and compare the obtained value with the one computed with the Bethe-Bloch formula (see problem 2).
Hint: assume $\beta = 1$, remember that $E_{kin} = m_o c^2 \cdot (\gamma - 1)$, and use $I = 16 \cdot Z^{0.9}$.

Problem 4 Energy loss of electrons in iron

The energy loss of electrons traveling through an absorber made of iron is described by

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_{ionization} + \left(\frac{dE}{dx}\right)_{radiation}$$

i.e. with a contribution of ionization losses and radiation losses.

a) Sketch qualitatively the dependence of the energy losses through ionization and radiation losses as function of the Lorentz factor $\gamma = E/m_e c^2$;

b) in this context, explain the meaning of the critical energy E_{krit}.

c) calculate the energy loss through ionization and radiation losses of an electron with an energy of 20 MeV in 1 cm iron; Calculate the critical energy for electrons in iron according to the emphirical relation given in the lecture.

d) compare the energy loss values found at point c) with the energy loss found in Problem 2 for a proton with the same kinetic energy.

Hint: Calculate the ionization energy loss according to the equation given in problem 2 above. The energy loss due to bremsstrahlung is given in the lecture as

$$\frac{dE}{dx} = \frac{4NZ}{A}\alpha r_e^2 E \left[\ln(191Z^{-1/3}) + \frac{1}{18}\right].$$

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Lecture web site: http://particle.astro.ru.nl/goto.html?partcos2021