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Particles and the Cosmos – 2020/21

Werkcollege 10 – Cosmic-ray propagation

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Problem 17 Leaky Box Model

The propagation of cosmic rays through the Galaxy is described using the transport equation

$$\frac{\partial N_i}{\partial t} = D \cdot \nabla^2 N_i + \frac{\partial}{\partial E}\left[b(E)N_i\right] + Q_i - \frac{N_i}{\tau_{spal}(i)} - \frac{N_i}{\tau_r(i)} + \sum_{j>i} \frac{p_{ij}}{\tau_j} N_j \quad .$$

a) Explain the physical meaning of the individual terms.

In the "Leaky Box Model" the diffusion term is replaced by $-N_i/\tau_e(i)$, where $\tau_e(i)$ is the escape time. Assume all isotopes of an element are produced exclusively through spallation and consider the state of equilibrium, i.e. $\partial N_i/\partial t = 0$. The production rate for isotope i is given as

$$C_i = \sum_{j>i} \frac{p_{ij}}{\tau_j} N_j$$

b) By neglecting the term of energy loss/energy gain of the particles, show that for stable isotopes

$$-\frac{N_i}{\tau_e(i)} + C_i - \frac{N_i}{\tau_{spal}(i)} = 0$$

and for radioactive isotopes with decay time $\tau_r(j)$

$$-\frac{N_j}{\tau_e(j)} + C_j - \frac{N_j}{\tau_{spal}(j)} - \frac{N_j}{\tau_r(j)} = 0.$$

c) By assuming $\tau_{spal} >> \tau_e$ and $\tau_{spal} >> \tau_r$, calculate the ratio of a radioactive to a stable isotope N_j/N_i in a state of equilibrium.

Problem 18 Age of cosmic rays

Beryllium in cosmic rays is most largely produced through spallation processes of heavier nuclei on the interstellar medium. Therefore, from the Leaky-Box model (see also Problem 17), the ratio of the aboundances of the two beryllium isotopes ^7Be (stable) and ^{10}Be (radioactive, $\tau_r(^{10}Be) = \gamma \cdot \tau_0 = 3.9 \cdot 10^6$ years) in cosmic rays is given as

$$\frac{N(^{10}\mathrm{Be})}{N(^{7}\mathrm{Be})} = \frac{\frac{1}{\tau_e(^7Be)}}{\frac{1}{\tau_e(^{10}Be)} + \frac{1}{\tau_r(^{10}Be)}} \frac{C(^{10}Be)}{C(^7Be)}$$

The ratio of the production rates is obtained from the corresponding cross sections as $C(^7Be)/C(^{10}Be) = 6.66$, while we can safely assume that $\tau_e(^7Be) = \tau_e(^{10}Be)$. The ratio $N(^{10}Be)/N(^7Be) = 0.028$ has been measured in cosmic rays. By assuming that $\tau_e = \tau_e(Be)$, calculate the residence time τ_e of cosmic rays in the Galaxy.

Problem 19 Matter traversed by cosmic rays

Start with the transport equation in Problem 17 and neglect diffusion as well as energy losses and energy gain during the propagation. Assume, the light elements Li, Be, and B (L group) are produced exclusively through spallation of the elements C, N, and O (M group).

Be $\xi = \rho \cdot x = \rho \cdot v \cdot t$ the path length of cosmic rays [g/cm^2], with the velocity of the particles v and the density of the interstellar medium ρ.

a) Derive the transport equations for the light nuclei

$$\frac{dN_L(\xi)}{d\xi} = -\frac{N_L(\xi)}{\xi_L} + \frac{P_{ML}}{\xi_M} N_M(\xi)$$

and for particles of the CNO group

$$\frac{dN_M(\xi)}{d\xi} = -\frac{N_M(\xi)}{\xi_M}.$$

The mean free path for collisions with the hydrogen atoms of the interstellar medium is $\xi_L = 8.4$ g/cm^2 for the L group and $\xi_M = 6$ g/cm^2 for the M group. $P_{ML} = 0.28$ is the mean fragmentation probability for nuclei of the M group.

b) Derive the differential equation

$$\frac{d}{d\xi}\left[e^{\frac{\xi}{\xi_L}} N_L(\xi)\right] = \frac{P_{ML}}{\xi_M} \exp\left(\frac{\xi}{\xi_L} - \frac{\xi}{\xi_M}\right) N_M(0)$$

and show that

$$\frac{N_L(\xi)}{N_M(\xi)} = \frac{P_{ML}\xi_L}{\xi_L - \xi_M}\left[\exp\left(\frac{\xi}{\xi_M} - \frac{\xi}{\xi_L}\right) - 1\right]$$

is a solution of this differential equation.

c) The relative abundances of cosmic rays has been measured $N_L(\xi)/N_M(\xi) = 0.25$. Calculate the amount of matter ξ [g/cm^2] traversed by cosmic rays on their way to the observer.

d) Cosmic rays are fully relativistic particles, so we can assume their velocity v to be equal to the speed of light c. By using the value of the cosmic-ray residence time in the galaxy τ_e as found in Problem 18, compute the average density of the medium crossed by the cosmic rays during their permanence in the galaxy. Compare the found value with the average interstellar medium density on the galactic plane $\rho_{disk} = 1$ particle cm^{-3}.

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Lecture web site: http://particle.astro.ru.nl/goto.html?partcos2021