Radboud Universiteit Nijmegen
Jörg R. Hörandel and Sascha Caron
Abha Khakurdikar

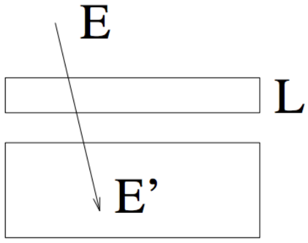
Particles and the Cosmos – 2020/21

Werkcollege 2 – Observables and experiments

09.09.2020

Problem 5 Silicon Detector as thin detector

Silicon detectors are often used for detecting cosmic-rays in association with particle calorimeters. The detection setup consists of a thin silicon detector and a thick detector (i.e the particle calorimeter), in which the particles are absorbed, see sketch.



Particles with kinetic energy E loose the energy ΔE in the thin detector. The remaining energy E' is measured with the thick detector, i.e. $\Delta E = E - E'$. In a generic material, nuclei with mass M and charge Z penetrate a distance R equal to

$$R_{Z,M}(E/M) = k\frac{M}{Z^2}\left(\frac{E}{M}\right)^{\alpha}$$

and the silicon detector thickness L can be written as

$$L = R_{Z,M}(E/M) - R_{Z,M}(E'/M) \ .$$

With this configuration, mass and charge of an atomic nucleus can be determined unambiguously.

Show that

$$M = \left(\frac{k}{Z^2 L}\right)^{1/(\alpha-1)} \left(E^{\alpha} - E'^{\alpha}\right)^{1/(\alpha-1)}$$

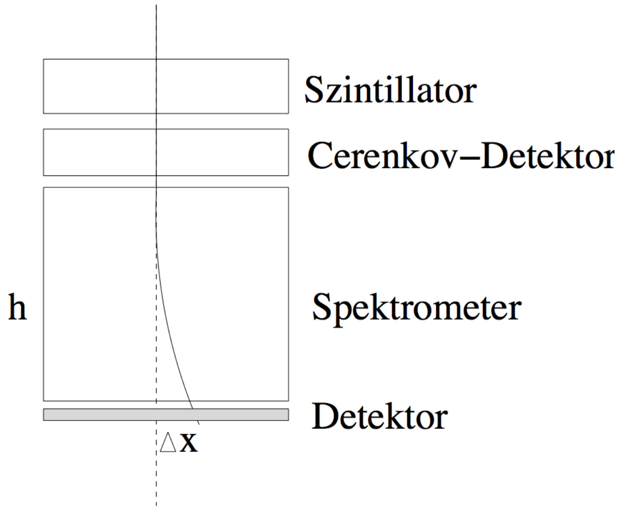
and

$$Z = \left(\frac{k}{L(2+\epsilon)^{\alpha-1}}\right)^{1/(\alpha+1)} \left(E^{\alpha} - E'^{\alpha}\right)^{1/(\alpha+1)}$$

Hint: assume $M/Z = 2+\epsilon$. This is assumption is valid for light nuclei with $Z < 30$.

Problem 6 Magnet Spectrometer

The momentum of a particle can be measured with a magnet spectrometer. Recent experiments usually comprise a silicon detector to measure the charge Z of the particles, a Čerenkov detector to measure the particle velocity $\beta = v/c$, and a magnet spectrometer. The latter measures the rigidity of the particles. The rigidity is given as $R = pc/(Ze)$, with the particle momentum p, the speed of light c and the elementary charge e.



Assume a magnet spectrometer with homogeneous magnetic field ($B = 1$ T), a height of $h = 1$ m, and a spatial resolution equal to $\Delta x = 200$ μm for measuring the particle trajectory.

a) Calculate the maximum momentum p_{max}, which can be measured for the given spatial resolution for a proton ($Z = 1$, $A = 1$) and a helium nucleus ($Z = 2$, $A = 4$). Express the result in the unit [GeV/c].
(1 GeV/c $= 5.34 \cdot 10^{-19}$ kg m/s.)

b) Charge Z and mass M can be measured with such a detector as well. The momentum is given as $p = M\beta\gamma c$ with the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. Show that

$$M = \frac{RZe}{c^2} \cdot \sqrt{\frac{1}{\beta^2} - 1}.$$

Problem 7 Electromagnetic calorimeter

Electromagnetic calorimeters are used to measure the energy of photons and $e^\pm$ through multiple bremsstrahlung and pair-production interactions, which create an electromagnetic shower composed of photons and $e^\pm$. For $e^\pm$ with an energy above 1 GeV, the radiation length, i.e. the average distance x that a particle needs to travel in a material to reduce its energy to $1/e$ of its original energy, can be expressed as

$$X_0 = \frac{716\ A}{Z(Z+1)\ \ln\left(287/\sqrt{Z}\right)}\ \text{g cm}^{-2}$$

The critical energy, defined as the energy at which the electron ionization losses and bremsstrahlung losses become equal, is given by

$$\epsilon = \frac{610}{Z+1.24}\ \text{MeV}.$$

The depth at which the electromagnetic shower reaches its maximum number of particles can be approximated as

$$X_{max} \simeq \ln\left(\frac{E}{\epsilon}\right) \cdot X_0.$$

The lateral extension of a shower is given by the Molière radius

$$R_M = \frac{21 \text{ MeV}}{\epsilon} \cdot X_0$$

a) Evaluate the radiation length for a calorimeter made of Tungsten ($A = 183.84$, $Z = 74$) and for one made of Carbon ($A = 12$, $Z = 6$);

b) Given an electromagnetic shower started by a 1 TeV electron, evaluate the required linear thickness and width of a calorimeter made of Tungsten and another one made of Carbon with the following requirements: the shower should reach X_{max} and has to be laterally confined. Discuss the obtained results.

Problem 8 Hadronic calorimeter

Similarly to electromagnetic calorimeters, hadronic calorimeters are absorbing detectors, aimed to measure the total energy of impinging particles. Usually they are made of several layers of passive absorber material alternated with sampling layers (e.g. scintillators, multi-wire chambers, proportional tubes, etc.). Compared to electromagnetic calorimeters, the thickness of the absorber layers is optimized to the hadronic interaction length, which is given as

$$\lambda = \frac{A}{N_A \cdot \rho \cdot \sigma_{tot}},$$

where A is the atomic mass number of the impinging particle, N_A is Avogadro's constant, ρ is the material density, and σ_{tot} is the total interaction cross-section, i.e. the sum of the elastic and inelastic cross-section.
Since σ_{tot} scales weakly with the particle energy for $\sqrt{s}$ in the range $1 - 100$ GeV and $\sigma_{tot} \propto A^{2/3}$, the formula can also be approximated by

$$\lambda = 35 \cdot A^{1/3} \text{ g cm}^{-2}.$$

a) Compute the interaction length of protons and the radiation length of electrons in a Tungsten calorimeter.
Hint: for computing the radiation length, use the formula given in Problem 7.

b) Given the result of problem a), discuss if a hadronic calorimeter can efficiently disentangle hadrons and leptons and if an electromagnetic calorimeter can.

c) In most collider experiments, like CMS and Atlas, the individual detectors are arranged in concentric shells around the interaction point. Usually, electromagnetic calorimeters are located in a shell closer to the interaction point than hadronic calorimeters. Discuss the possible reason.

Student assistant: Abha Khakurdikar email: abha.khakurdikar at student.ru.nl
Lecture web site: http://particle.astro.ru.nl/goto.html?partcos2021