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Particles and the Cosmos – 2020/21

Tutorial 8 – Introduction to Astroparticle Physics

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Problem 9 Mean free path of solar neutrinos

The cross section for neutrinos with energy E_{cm} (center of mass system) for inelastic interactions with nucleons is given as

$$\sigma_{\nu-n} = 5 \cdot 10^{-44} \left(\frac{E_{cm}}{1 \text{ MeV}} \right)^2 \text{ cm}^2.$$

a) Calculate the mean free path $\lambda = 1/(\sigma_{\nu-n} n)$, with the number density n for neutrino capture of neutrinos with a lab energy of 1 MeV in the center of the Sun (typical density $\langle \rho \rangle \approx 100 \text{ g/cm}^3$).
Hint: discuss the difference for the given case between center of mass and laboratory reference frame.

b) Compare the result obtained to the radius of the Sun $R_{Sun} = 7 \cdot 10^5$ km.

Problem 10 Neutrino mass estimation from SN 1987 A

Neutrinos from supernova SN 1987 A were observed with the Kamiokande detector. The following neutrinos were observed:

event no	$t_{obs}(s)$	neutrino energy (MeV)
1	0	21.3
2	0.107	14.8
3	0.303	8.9
4	0.324	10.6
5	0.507	14.4
6	1.541	36.9
7	1.728	22.4
8	1.915	21.2

The observation times are given relative to the detection of the first neutrino.

a) Knowing the distance of the supernova is 52 kpc, calculate the time t_0 it takes for the neutrinos to reach Earth assming they travel with the speed of light.

b) Show that a neutrino with mass m and energy E will take a total time of

$$\Delta t = t_{obs} - t_{em} = t_0 \left(1 + \frac{m^2}{2E^2} \right)$$

to reach the Earth ($c = 1$ convention is used).

c) According to astrophysical models the neutrinos were emitted within an interval of 2 s. Derive an upper limit for the neutrino mass from the data listed in the table. Assume all neutrinos have the same mass.

(This problem is based on W.D. Arnett and J.L. Rosner Phys. Rev. Lett. 18 (1987) 1906. This method provided at the time of publication one of the best upper limits for the neutrino mass.)

Problem 11 Power law

In astrophysics quantities often follow power laws. As example we consider a differential energy spectrum

$$\frac{dN}{dE} = N_0 E^{\gamma} \quad \left[\frac{1}{\mathrm{eV}\cdot\mathrm{m}^2\cdot\mathrm{s}\cdot\mathrm{sr}}\right].$$

The differential spectrum gives the number of particles per energy, per unit area, per unit time, and per unit solid angle.

The particle flux above a certain energy value E_0 is given as

$$N_{>E_0} = \int_{E_0}^{\infty} N_o E^{\gamma} dE \quad \left[\frac{1}{\mathrm{m}^2\cdot\mathrm{s}\cdot\mathrm{sr}}\right].$$

By assuming that for $E_{th} = 10^{15}$ eV the integral flux of particles from the upper hemisphere (i.e. solid angle $\Omega = 2\pi$ sr) is

$$N_{>E_{th}} = 1 \text{ particle} \cdot \mathrm{m}^{-2} \cdot \mathrm{year}^{-1}$$

and that the power law indices γ is equal to

$$\gamma = \begin{cases} -2.7, & \text{if } E < E_{th} \\ -3.0, & \text{if } E > E_{th} \end{cases}$$

compute the integral flux $N_{>E}$ for $E_1 = 10^9$ eV and $E_2 = 10^{18}$ eV, respectively.

Problem 12 Energy density of a magnetic field

The energy density (i.e. the energy E per volume V) of a magnetic field in a generic medium is given as

$$\epsilon_B = \frac{B^2}{2\,\mu_0\,\mu_r}.$$

a) Calculate the energy density ϵ_B of a magnetic field $B = 3\ \mu$G in vacuum, i.e. $\mu_r = 1$. Give the result in [eV/cm^3].

b) Compare the obtained result with the starlight energy density in the solar neighborhood $\epsilon_{SL} = 0.43$ eV cm^{-3} as estimated by Mathis, Mezger, and Panagia on 1982.

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