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Particles and the Cosmos – 2020/21

Werkcollege 9 – Introduction to Astroparticle Physics

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Problem 13 Stochastic acceleration

Relativistic protons are accelerated at a shock front. At each crossing of the shock the particles gain $\xi = 20\%$ energy. The probability to again cross the shock is $P = 80\%$.

Derive the form of the energy spectrum and calculate the spectral index γ.

Problem 14 Second order Fermi acceleration

Particles are accelerated at magnetic clouds with a speed $V = 10^{-4}$ c. Calculate the time needed to accelerate particles form an energy of 100 MeV to 1 PeV. The diffusion coefficient of cosmic rays in the Galaxy is $D = 10^{28}$ cm^2/s.

Compare this time to the confinement time of cosmic rays in the Galaxy.

Consider the first interaction of a cosmic ray particle with a cloud. Calculate the energy gain in the first interaction and compare it to the energy loss through ionization in the interstellar medium. Assume $\rho = 1$ H/cm^3 and $dE/dx = 2$ MeV/(g/cm^2).

Hint: the mean free path between two clouds can be estimated from D.

Problem 15 Gas diffusion

Consider an ideal gas in a box. The box is divided in two halfs by a wall with area A, and on one side we have the mass density ρ_1 on the other side ρ_2. When the separation wall is removed, diffusion starts until an equilibrium of the gas densities is reached.

A mass flow per unit time emerges along the x-axis

$$\frac{\Delta m}{\Delta t} = -D \cdot A \cdot \frac{d\rho}{dx},$$

with the diffusion coefficient D[cm^2/s] and the density gradient $d\rho/dx$.

Given the Avogadro constant N_A and the molar mass M (in case of a mixed gas, M is the average molar mass), the current density is given as

$$j = \frac{\Delta m}{A \cdot \Delta t} \cdot \frac{N_A}{M},$$

and the particle number density is given as

$$n = \frac{\rho \cdot N_A}{M}.$$

Show that for this case

$$\vec{j} = -D \cdot \frac{\partial n}{\partial x} \qquad 1^{st} \text{ law of Fick}$$

and

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial x}\left(D \cdot \frac{\partial n}{\partial x}\right) \quad 2^{nd} \text{ law of Fick.}$$

Problem 16 Mean Free Path

Cosmic-ray particles move through the Galaxy. Estimate the mean free path of cosmic-ray particles between two collisions with particles of the interstellar medium. Assume a particle density of the interstellar medium of 1 proton/cm^3 ($m_p = 1.67 \cdot 10^{-24}$ g).

Use the geometrical cross section with a radius $r_A = r_0 A^{1/3}$ for nuclei with mass number $A > 1$ ($r_0 = 1.3$ fm) and $r_p = 0.8$ fm for protons (1 fm=10^{-15} m).

Calculate the mean free path for protons, oxygen nuclei, and iron nuclei. Give the result as column density [g/cm^2].

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