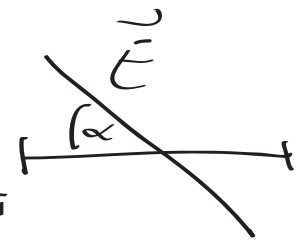
Antenna temperature

power density received by an antenna

$P_\nu = A_{eff} \cdot S_\nu$ $\quad A_{eff}$ effective area of antenna

net power collected by an antenna (unpolarized radiation)

dipole integrating over all directions $\alpha = 0 .. 2\pi$



$$E(\alpha) = E_0 \cdot \cos\alpha$$

$$\langle P \rangle = \frac{E_0^2}{2\pi} \int_0^{2\pi} \cos^2\alpha \, d\alpha = \frac{E_0^2}{2\pi} \left(\frac{\alpha}{2} + \frac{1}{4} \sin 2\alpha \right) \Big|_0^{2\pi}$$

$$= \frac{E_0^2}{2}$$

integration over whole bandwidth $\Delta\nu$ of receiver

$$P = \frac{1}{2} \cdot A_{eff} \cdot S_\nu \cdot \Delta\nu$$

power received by a simple dipole antenna

next: antenna temperature

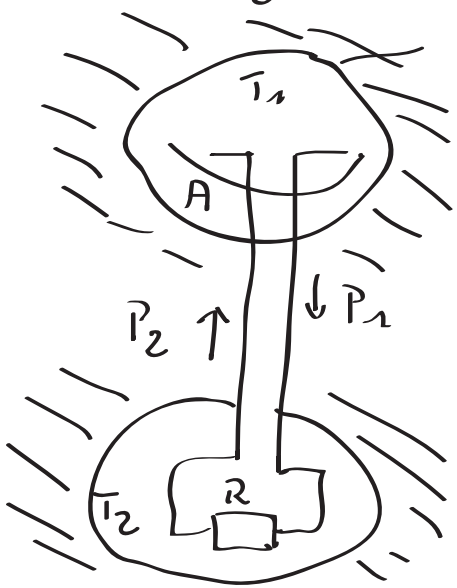
→ „the physical connection from the telescope to the sky"

needed for calibration of the flux density

antenna is enclosed by black body with T_1

thermal equilibrium

$\Rightarrow P_1 = P_2$



radiation density of black body is given by Planck's law

$$u_\nu \, d\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} \, d\nu$$

the corresponding intensity is given as

$$B_\nu \, d\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \, d\nu$$

$h\nu \ll kT \rightarrow$ Rulegh-Jeans approximation

$$B_\nu \, d\nu = \frac{2kT}{\lambda^2} \, d\nu$$

corresponding deviation of the noise power flowing in transmission line connected to a black body at temperature T

$\rightarrow$ one-dimensional Planck law

$$P_\nu \, d\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \, d\nu \xrightarrow[\text{Jeans}]{\text{Rayleigh}} \boxed{P_\nu \, d\nu = kT \, d\nu}$$

Nyquist law: resistor at temperature T produces noise power $kT \, d\nu$ per unit band width

equating this with incomming flux density S_ν

$$k \cdot T_A \cdot \Delta\nu = \frac{1}{2} A_{eff} \cdot S_\nu \cdot \Delta\nu$$

$$\rightarrow \boxed{S_\nu = \frac{2k}{A_{eff}} \cdot T_A}$$

this relates the flux density of a source to an antenna temperature

it depends on the collecting area of the telescope

Antenna gain

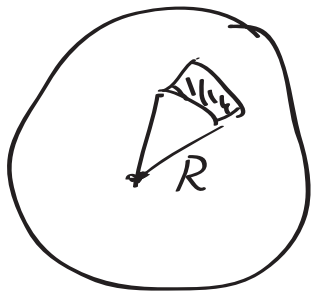
an antenna is characterized by a property to quantify its ability to measure radiation from a particular direction

$G(\vartheta, \varphi)$ antenna gain

it is defined as

$$G = \frac{4\pi R^2 S_\nu}{P_\nu}$$

S_ν flux density
P_ν power
R distance



isotropic radiator $G = 1$

gain depends on direction, therefore

$$A_{eff} = A_{eff}(\vartheta, \varphi)$$

$$A_{eff} = \varphi \cdot G(\vartheta, \varphi)$$ φ will be determined below

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$

The gain G tells us how much power an isotropic radiator must radiate such as to produce the same flux density at some distance R as if the same flux density would be produced in that direction using an antenna having this gain G

Antenna temperature and brightness temperature

the connection between brightness or intensity $B_\nu(T_b)$ and the corresponding radiation or brightness temperature T_b

→ relationship between antenna temperature T_A and brightness temperature T_b, closely related to physical parameters we want to measure

again Planck's law in Rayleigh Jeans approximation

$$B_\nu = \frac{2\nu^2 k}{c^2} T_b \quad \text{or} \quad B_\nu = \frac{2k}{\lambda^2} T_b \qquad (1)$$

relation between flux density and brightness

$$S_\nu = \int_0^{\pi} \int_0^{2\pi} B_\nu(\vartheta, \varphi)\, d\Omega \qquad [B_\nu] = \frac{W}{m^2\, Hz\, sr}$$

$$d\Omega = \sin\vartheta\, d\vartheta\, d\varphi \qquad \text{(spherical coordinates)}$$

sr sterradian $= \text{deg} \times \text{deg}$

with (1):

$$S_\nu = \frac{2k}{\lambda^2} \int_0^{\pi} \int_0^{2\pi} T_b(\vartheta, \varphi)\, d\Omega \qquad \text{(Rayleigh-Jeans)}$$

$$= \frac{2k}{A_{eff}}\, T_A$$

$$\Rightarrow T_A = \frac{A_{eff}}{\lambda^2} \cdot \int_0^{\pi} \int_0^{2\pi} T_b(\vartheta, \varphi)\, d\Omega$$

relation between antenna temperature and brightness temperature

Antenna pattern

The antenna pattern describes the sensitivity of an antenna $G(\delta, \varphi)$

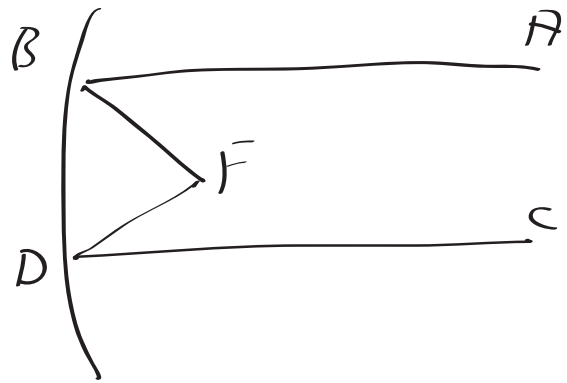
G can be measured using a point source at angles δ, φ or it can be derived theoretically

parabolic mirror

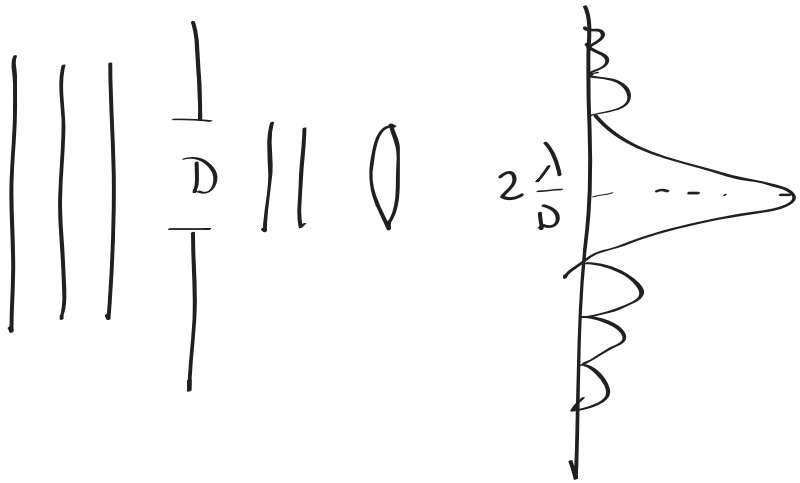
focus point F

aperture AC

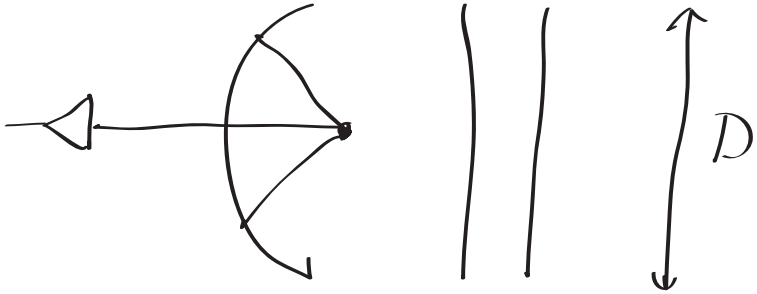
$$AB + BF = CD + DF$$

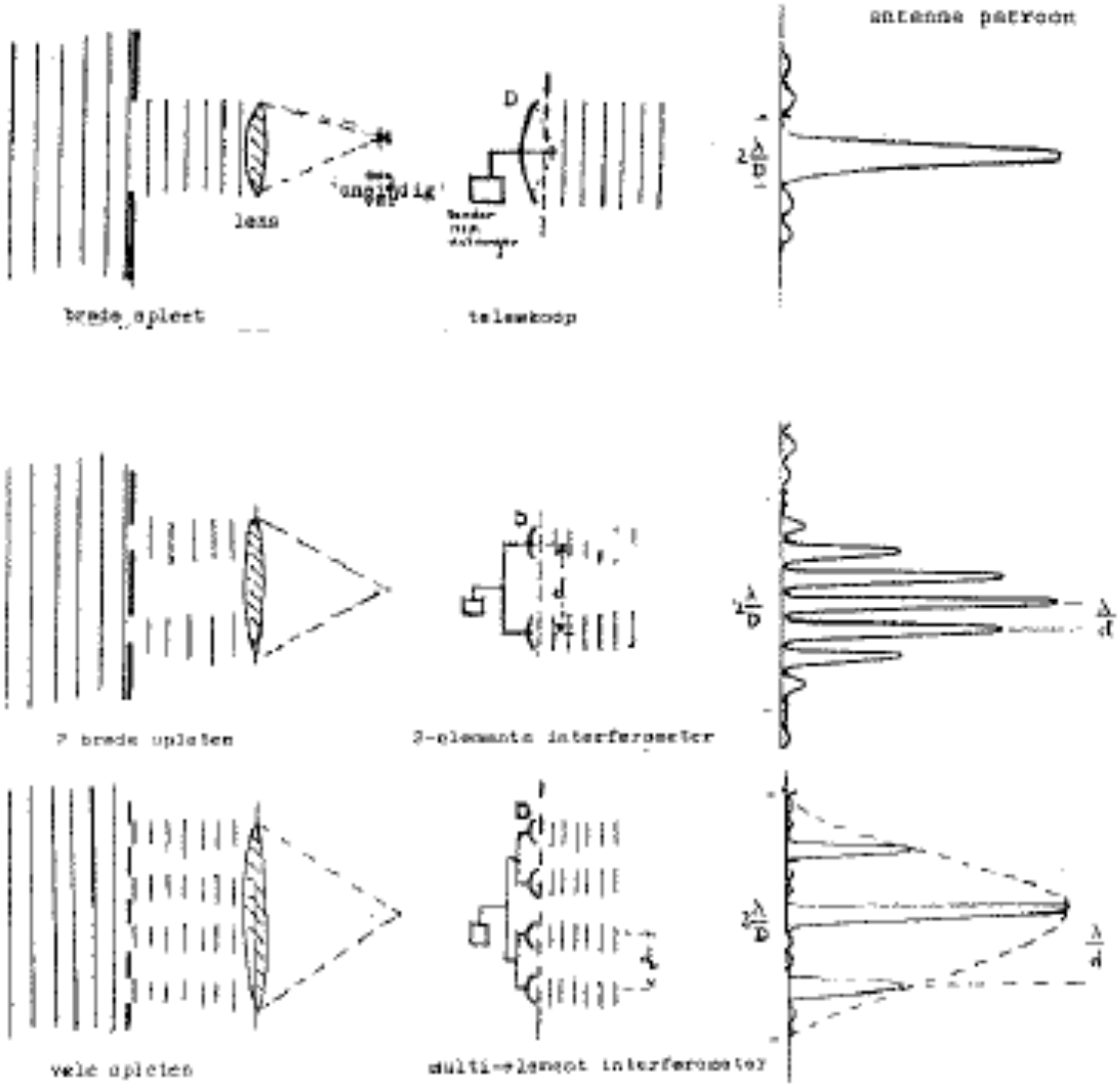


remember optics, diffraction of light at a single slit



correspondingly, telescope

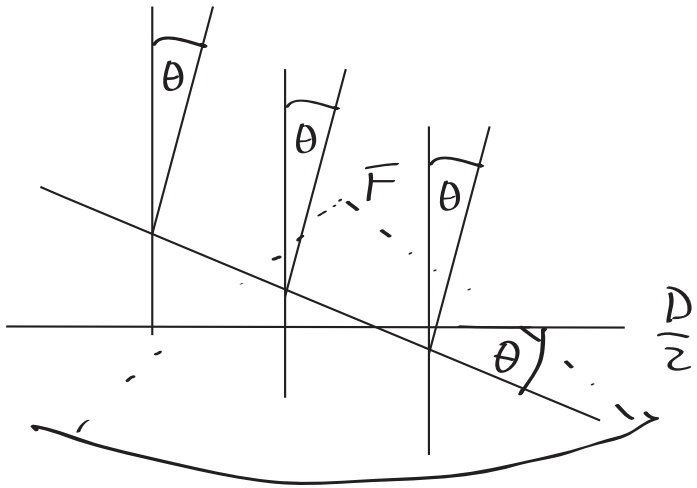




Figuur 3.3: Analogie van interferometer met vlakke golf op meerdere spleten

Antenna pattern of a simple telescope

telescope sees source at angle θ



electric field $E_\theta(t) = \sum_{i=1}^{N} E_n(t)$

$$E_n(t) = E_0 \cos(\omega t + \Delta\phi_n)$$

$$\Delta\phi_n = \frac{\Delta l}{\lambda} 2\pi$$

path length difference $\Delta l = \frac{n - \frac{N}{2}}{N} \cdot D \cdot \sin\theta$

$$\rightarrow \Delta\phi_n = \left(n - \frac{N}{2}\right) \frac{2\pi D}{\lambda N} \sin\theta$$

$\Sigma \rightarrow \int$ infinitisimal element

$$x = \left(n - \frac{N}{2}\right)\frac{D}{N} \qquad \Delta\phi(x) = \frac{x}{\lambda}\, 2\pi \sin\theta$$

$$E_\theta(t) = \int E_x(t)\,dx = E_0 \int_{-D/2}^{D/2} \cos(\omega t + \Delta\phi(x))\,dx$$

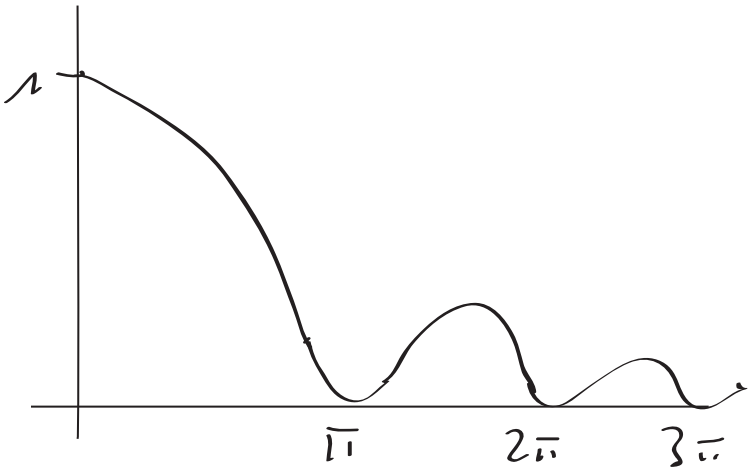
$$= E_0 \int_{-D/2}^{D/2} \cos\left(\omega t + 2\pi \frac{x}{\lambda} \sin\theta\right) dx$$

$$E(t) = E_0 \cos\omega t \, \frac{\sin\left(\pi \frac{D}{\lambda} \sin\theta\right)}{\frac{\pi}{\lambda}\sin\theta} \qquad P = \pi \frac{D}{\lambda} \sin\theta$$

$$E(t) = D\, E_0 \cos\omega t \, \frac{\sin P}{P}$$

and $V = \overline{E^2(t)} = V_0 \dfrac{\sin^2 p}{p^2}$ $\qquad V_0 = \dfrac{D^2}{2} E_0^2$

normalized to 1 at center

$$G(\theta) = \frac{\sin^2 p}{p^2}$$ one dimensional antenna pattern

$$\lim_{p \to 0} \frac{\sin^2 p}{p^2} = 1$$