Non-thermal emission

Synchrotron radiation

relativistic ($v \approx c$) charged particles which are deflected in a magnetic field emit synchrotron radiation

Lorentz force $\vec{F}_L = q\,\vec{v} \times \vec{B}$

q charge

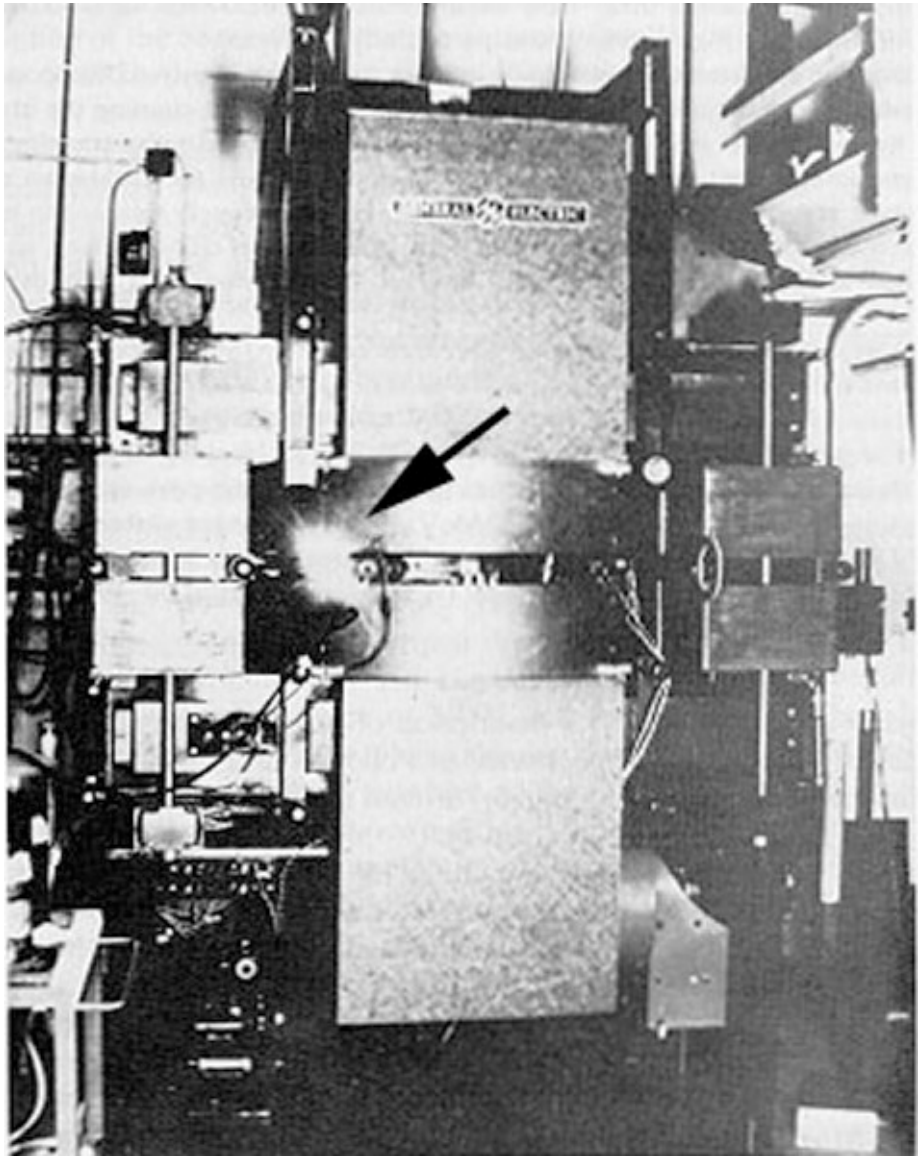
$\vec{v}$ velocity

$\vec{B}$ magnetic field

homogeneous B field $B \perp v$

centrepetal force $F_C = \frac{mv^2}{R}$ m mass

particles describe circle with radius $R = \frac{mv}{qB}$



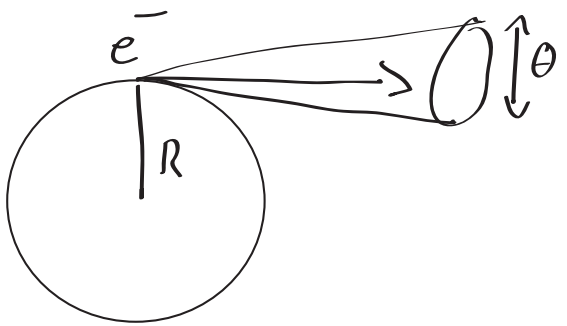
General Electric synchrotron accelerator built in 1946, the origin of the discovery of synchrotron radiation.

particle emits radiation with frequency

$$\nu = \frac{v}{2\pi R} = \frac{q B}{2\pi m}$$

electron on circle emits radiation directed perpendicular to direction of motion with power pattern of dipole transformed into laboratory frame

opening angle of radiation $\Theta = \frac{2}{\gamma}$



Lorentz factor $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

total particle energy $E = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $\quad p = mv$ momentum

$$\nu = \frac{e B}{2\pi m_0} \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

classical approximation $\Delta t \approx \frac{\theta}{2\pi\nu}$ time to look into light cone

relativistic case: contraction factor $\frac{1}{\gamma}$

in addition, correction of B-field $\frac{1}{\gamma}$

$$\Delta t \approx \frac{2 m_0}{e B} \frac{1}{\gamma^2}$$

frequency of radiation: $\nu_m = \frac{1}{2\pi \Delta t} \approx \frac{1}{4\pi} \frac{e B}{m_0} \gamma^2$

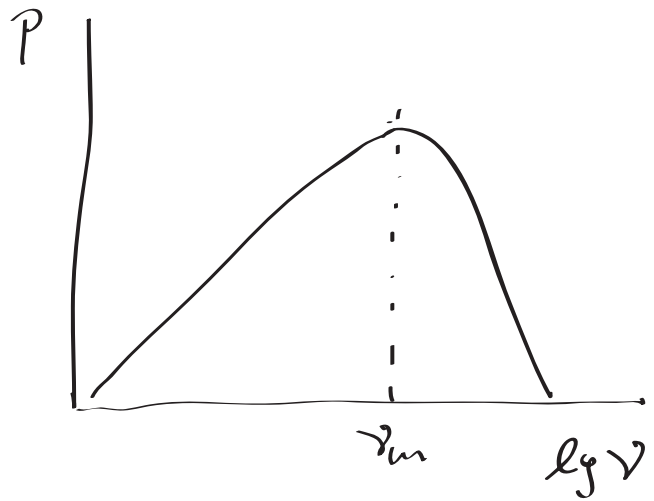
$$\nu_m = \frac{1}{4\pi} \frac{e B \varepsilon^2}{m_0^3 c^4} = 6 \cdot 10^{-6} B \varepsilon^2 \, Hz$$

B [G]

ε [eV]

exact relativistic derivation yields

$$I \propto \mathcal{E}^2 B^2 = \int W(\nu)\, d\nu \qquad W(\nu) \text{ emission spectrum}$$



Spectral distribution of synchrotron radiation

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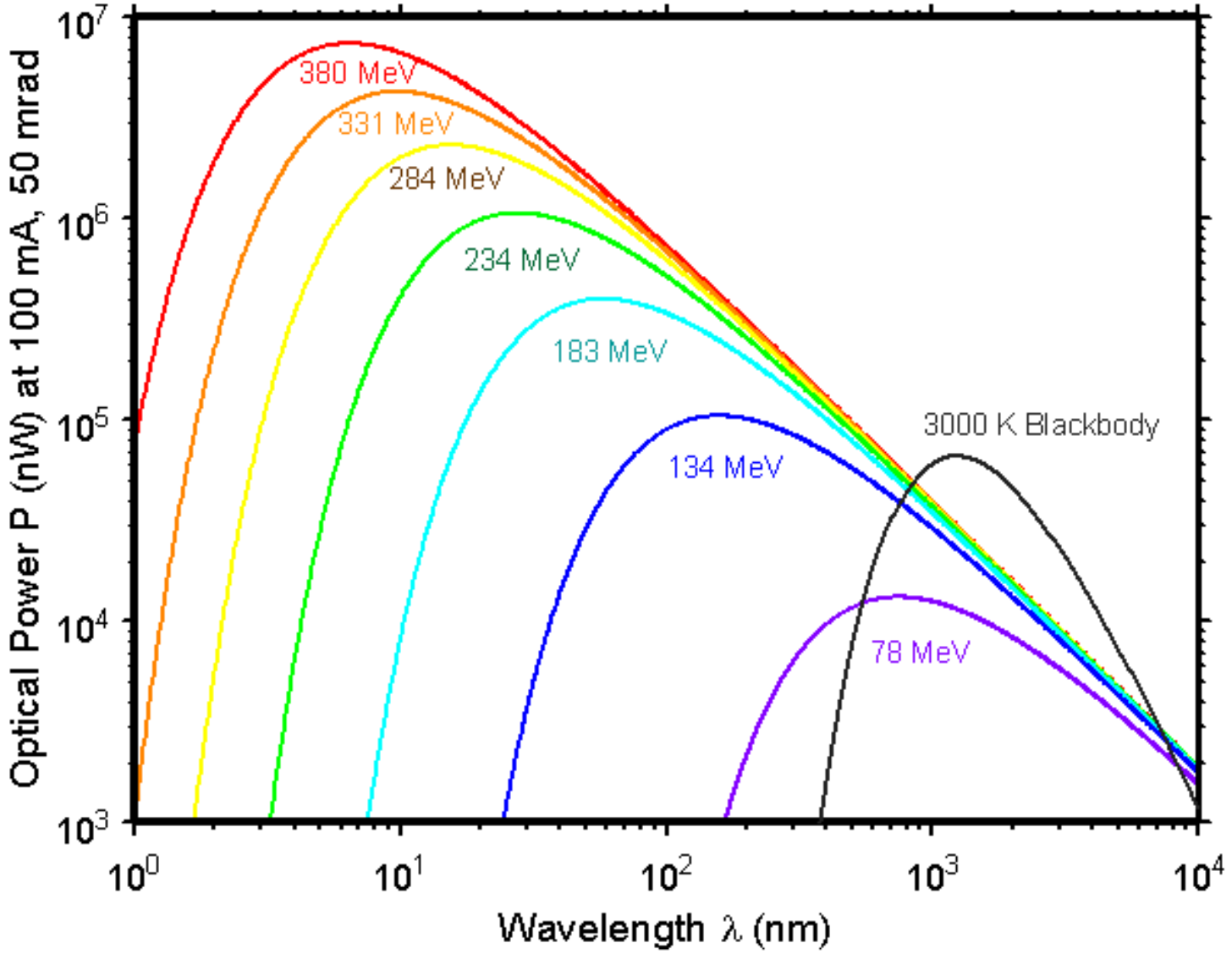
distribution function $n(E)\,dE$ gives number of particles with energies between E and $E+dE$

Synchrotron radiation spectrum



Cosmic rays $n(\mathcal{E})\,d\mathcal{E} = n_0\,\mathcal{E}^{-\gamma}\,d\mathcal{E}$

power emitted by electrons with energies $\mathcal{E}$ - $\mathcal{E}+d\mathcal{E}$

$P(\mathcal{E})\,d\mathcal{E} = n(\mathcal{E})\,I(\mathcal{E})\,d\mathcal{E}$ $\quad$ $I(\mathcal{E})$ intensity

radiation is emitted around ν_m in frequency band $d\nu$

$$P(\mathcal{E})\,d\mathcal{E} \propto \mathcal{E}^{-\gamma}\,\mathcal{E}^2\,B^2\,d\mathcal{E}$$

$$\nu_m \propto B\,\mathcal{E}^2$$

$$\rightarrow \quad d\mathcal{E} \propto \frac{d\nu}{B\,\mathcal{E}}$$

$$\text{and} \quad P(\nu)\,d\nu \propto \mathcal{E}^{1-\gamma}\,B\,d\nu = \left(\frac{\nu_m}{B}\right)^{\frac{1-\gamma}{2}} B\,d\nu$$

$$P(\nu)\,d\nu \propto (\nu_m)^{\frac{1-\gamma}{2}}\,B^{\frac{1-\gamma}{2}}\,d\nu$$

flux density of synchrotron source

$$S(\nu) \propto N_0 \, P(\nu) \propto N_0 \, \nu^{\frac{1-\gamma}{2}} \, B^{\frac{1+\gamma}{2}} \propto \nu^{\alpha}$$

$\alpha = \frac{1}{2}(1-\gamma)$ spectral index

—> investigate shape of the distribution of relativistic electrons from their measured synchrotron spectrum

polarization of synchrotron radiation

Synchrotron radiation is polarized (rather long and complicated derivation)

components parallel and perpendicular to B field

$$P_{\perp} = \frac{\sqrt{3}}{2} \frac{e^3 B \sin\alpha}{m c^2} \left(F(x) + G(x) \right) \qquad F(x) = x \int_x^{\infty} K_{\frac{5}{3}}(t)\, dt$$

$$P_{\parallel} = \frac{\sqrt{3}}{2} \frac{e^3 B \sin\alpha}{m c^2} \left(F(x) - G(x) \right) \qquad G(x) = x K_{\frac{2}{3}}(x)$$

K are modified Bessel functions $\qquad x = \frac{\nu}{\nu_c}$

$$\nu_c = \frac{3}{2} \gamma^2 \frac{eB}{mc} \sin\alpha \qquad \tan\alpha = \frac{|v_{\perp}|}{|v_{\parallel}|}$$

velocity components relative to B field

F(x) and G(x)

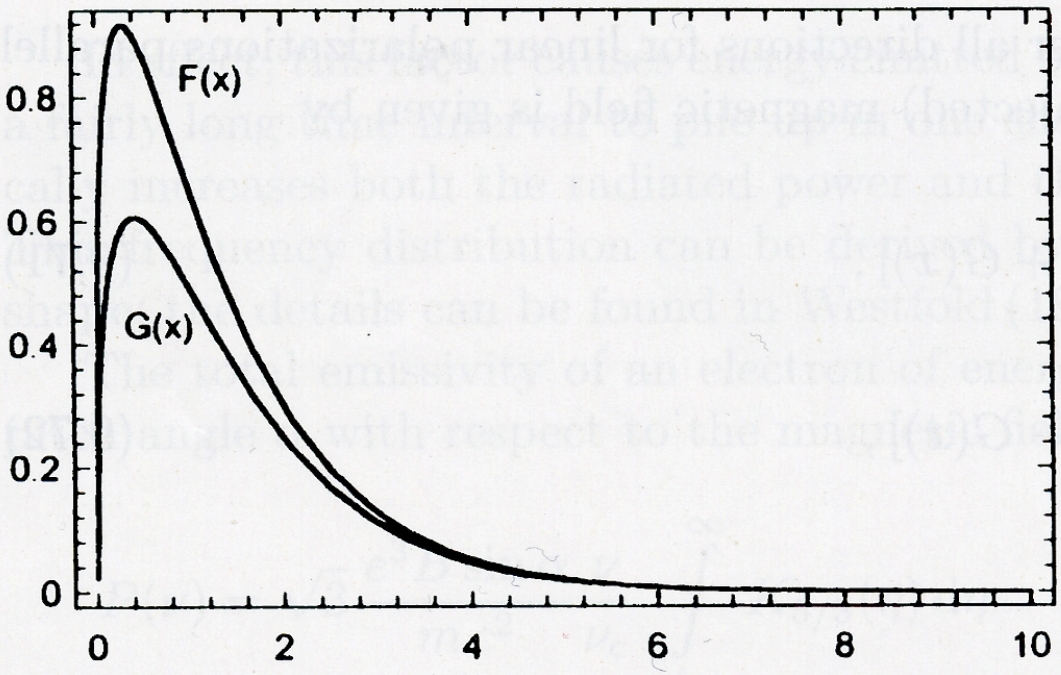


Fig. 9.8. The spectral distribution of the power of synchrotron radiation. The functions $F(x)$ and $G(x)$ are related by (9.71) and (9.72) to the linear polarization components parallel and perpendicular to the magnetic field, the frequency (9.75) is normalized by (9.76)

F(x) and G(x)

Table 9.1. Spectral Distribution of the emission from a charged particle moving in a magnetic field

x	$F(x)$	$G(x)$	p	x	$F(x)$	$G(x)$	p
0.00	0.0000	0.0000	0.500	1.00	0.6514	0.4945	0.759
0.01	0.4450	0.2310	0.519	1.10	0.6075	0.4669	0.769
0.02	0.5472	0.2900	0.530	1.20	0.5653	0.4394	0.777
0.03	0.6136	0.3305	0.539	1.30	0.5250	0.4123	0.785
0.04	0.6628	0.3621	0.546	1.40	0.4867	0.3859	0.793
0.05	0.7016	0.3881	0.553	1.50	0.4506	0.3604	0.800
0.06	0.7332	0.4102	0.560	1.60	0.4167	0.3359	0.806
0.07	0.7597	0.4295	0.565	1.70	0.3849	0.3125	0.812
0.08	0.7822	0.4465	0.571	1.80	0.3551	0.2904	0.818
0.09	0.8015	0.4617	0.576	1.90	0.3274	0.2694	0.823
0.10	0.8182	0.4753	0.581	2.00	0.3016	0.2502	0.829
0.12	0.8454	0.4988	0.590	2.50	0.1981	0.1682	0.849
0.14	0.8662	0.5184	0.598	3.00	0.1286	0.1112	0.865
0.16	0.8822	0.5348	0.606	3.50	0.0827	0.07256	0.877
0.18	0.8943	0.5486	0.613	4.00	0.0528	0.04692	0.888
0.20	0.9034	0.5604	0.620	4.50	0.0336	0.03012	0.897
0.22	0.9099	0.5703	0.627	5.00	0.0213	0.01922	0.904
0.24	0.9143	0.5786	0.633	5.50	0.0134	0.01221	0.910
0.26	0.9169	0.5855	0.639	6.00	0.00837	0.00773	0.916
0.28	0.9179	0.5913	0.644	6.50	0.00530	0.00487	0.920
0.30	0.9177	0.5960	0.649	7.00	0.00332	0.00306	0.923
0.40	0.9019	0.6069	0.673	7.50	0.002076	0.00192	0.926
0.50	0.8708	0.6030	0.692	8.00	0.001298	0.00120	0.927
0.60	0.8315	0.5897	0.709	8.50	0.000812	0.000752	0.926
0.70	0.7879	0.5703	0.724	9.00	0.000507	0.000469	0.924
0.80	0.7424	0.5471	0.737	9.50	0.0003177	0.0002920	0.919
0.90	0.6966	0.5214	0.749	10.0	0.0001992	0.0001816	0.912

linear polarization of synchrotron radiation averaged over all directions is remarkably high

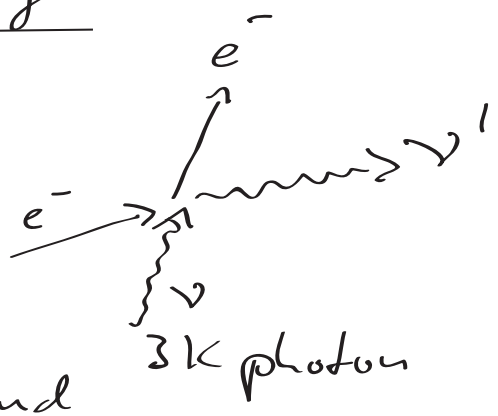
$$P = \frac{P_\perp - P_{\parallel}}{P_\perp + P_{\parallel}} = \frac{G(x)}{F(x)} = \begin{cases} 0{,}5 & x = 0 \\ 1{,}0 & x \to \infty \end{cases}$$

Inverse Compton Scattering

energetic electrons

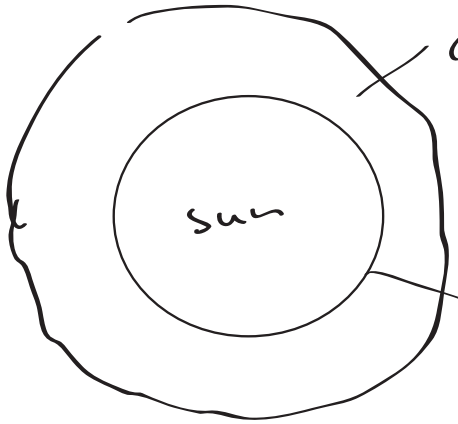
transfer energy

to photons of 3K background



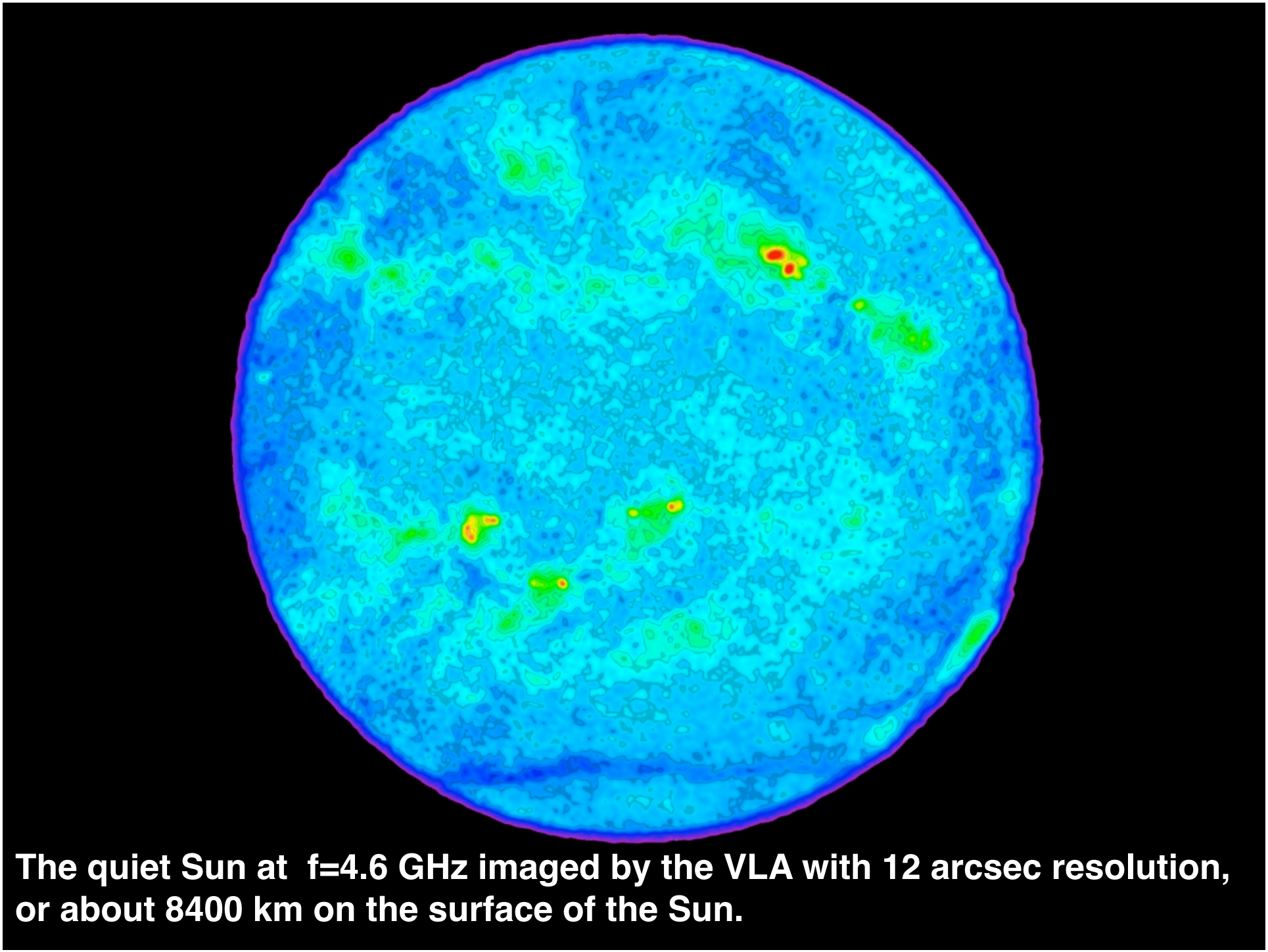
The Sun

closest and brightest source, thermal emission



low gas density in corona
→ no strong visual emission, but strong radio source

gas in corona fully ionized, constant electron temperature $T_e \sim 10^6$ K for $r - r_0 > 2 \cdot 10^9$ cm



The quiet Sun at f=4.6 GHz imaged by the VLA with 12 arcsec resolution, or about 8400 km on the surface of the Sun.

electron density distribution described by Baumbach - Allen formula

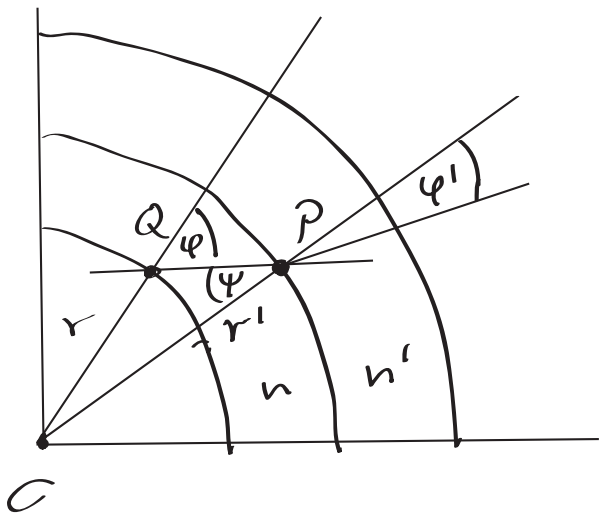
$$\frac{N_e}{cm^3} = \left[1{,}55 \left(\frac{r}{r_0}\right)^{-6} + 2{,}99 \left(\frac{r}{r_0}\right)^{-16} \right] 10^8$$

$r_0 = 7 \cdot 10^{10}$ cm solar radius

for calculation of optical depth, refraction plays an important role

radio waves propagate along curved paths

Corona is considered to consist of concentric shells of constant index of refraction



law of refraction

$n' \sin\varphi' = n \cdot \sin\varphi$

triangle CPQ: $\frac{\sin\psi}{r} = \frac{\sin(180^0 - \varphi)}{r'}$

$= \frac{\sin\varphi}{r'}$

$\rightarrow n \cdot r \cdot \sin\varphi = n' r' \sin\varphi' = \rho = \text{const}$

parameter ρ of the ray is equal to the minimum distance from the solar center that a straight line tangent to the exterior part of the ray would have attained

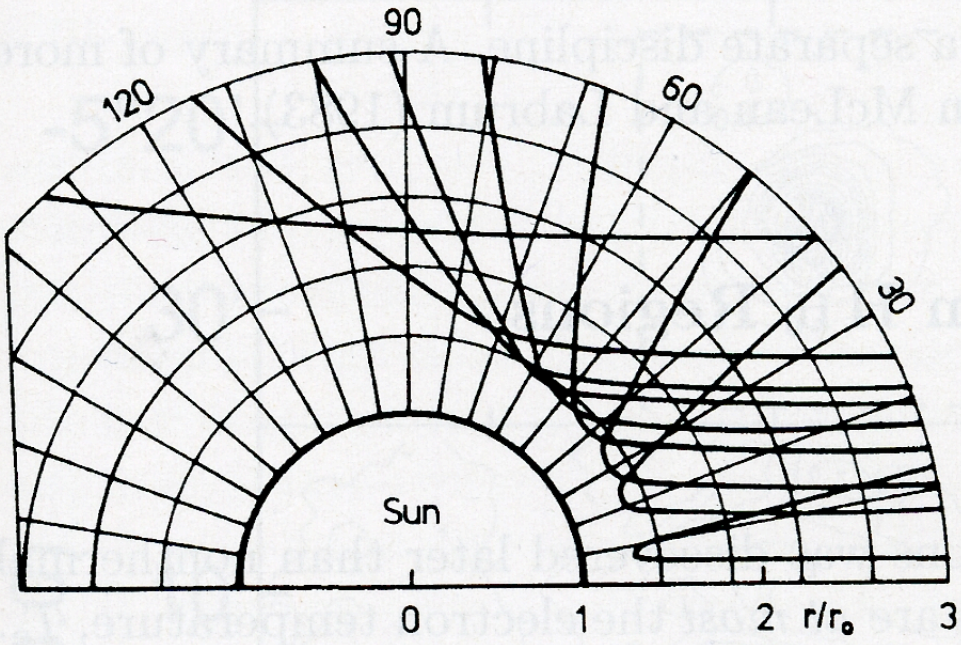


Fig. 10.3. Ray geometry in the solar atmosphere [after Jaeger and Westfold (1949)]

$\varphi(r)$ angle between ray and solar radius

differential equation for rays in solar corona

$$n(r)\, r \sin\varphi(r) = \varsigma$$

to calculate brightness distribution, equation of radiation transfer has to be solved along trajectories

each ray has a point of closest approach: $\varphi = \frac{\pi}{2}$

brightness distribution across the solar disc depends on optical depth along ray in relation to point of closest approach

1) low frequencies $\nu < 0{,}1$ GHz ($\lambda > 3$ m)

strong refraction

point of closest approach well above layers with $\tau = 1$

brightness decreases with r

$\tau \ll 1$: brightness temperature is less than corona temperature

2) 0.1 GHz $< \nu < 3$ GHz

$\tau \approx 1$ near point of closest approach

refraction can be neglected

luminous atmosphere above non-luminous stellar body → light rim

3) $\nu > 3$ GHz $\lambda < 0.1$ m

optical thickness in corona is too small to produce strong radiation

→ no light rim

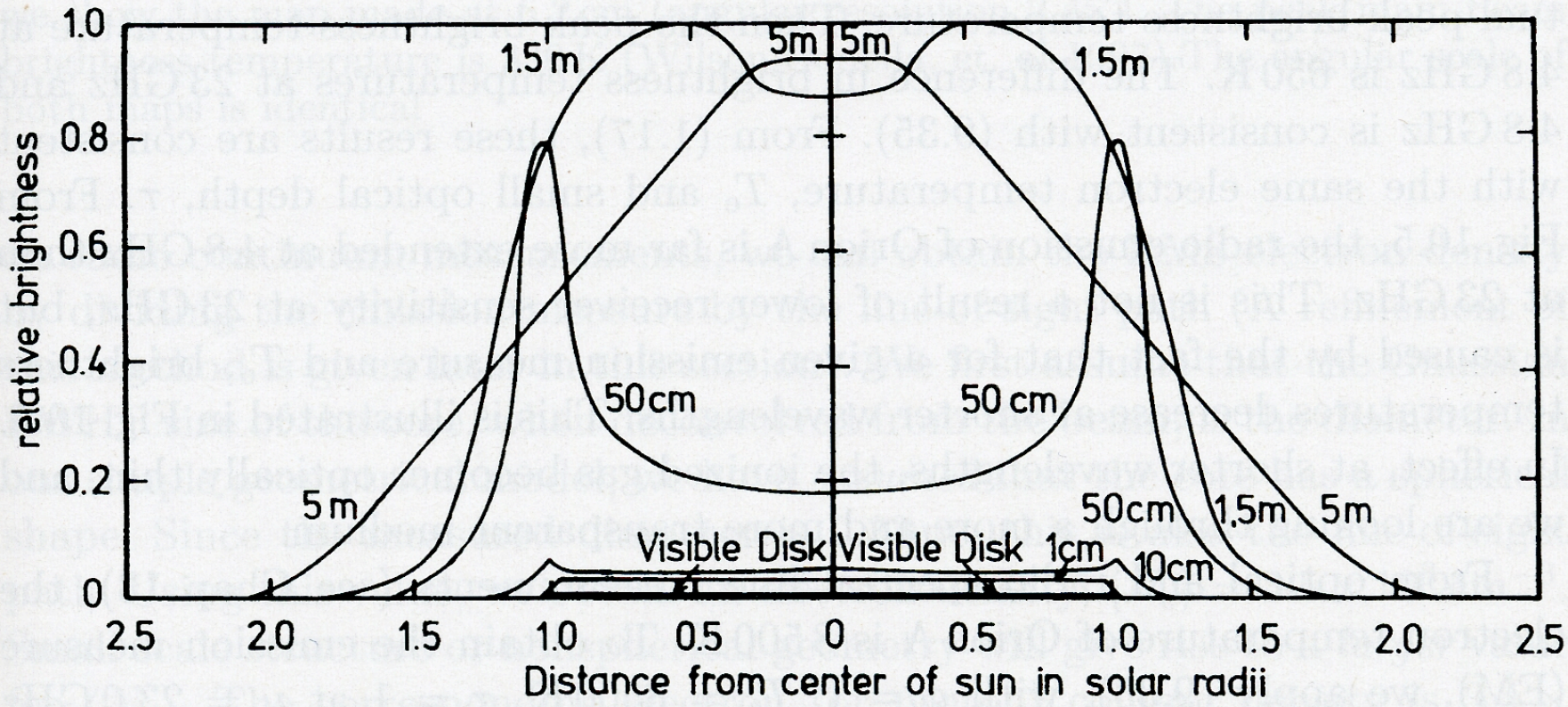


Fig. 10.4. Brightness distribution across the solar disk at different wavelengths [after Smerd (1950)]