On the point-source approximation of nearby cosmic ray sources

Satyendra Thoudam* and Jörg R. Hörandel

Department of Astrophysics, IMAPP, Radboud University Nijmegen, PO Box 9010, 6500 GL Nijmegen, the Netherlands

Accepted 2011 August 29. Received 2011 August 23; in original form 2011 June 27

ABSTRACT

In this paper, we check in detail the validity of the widely adopted point-source approximation for nearby cosmic ray (CR) sources. Under an energy-independent escape model for CRs from the sources, we show that for young nearby sources, the point-source approximation breaks down at lower energies and the CR spectrum depends on the size and the morphology of the source. When applied to the nearby supernova remnants (SNRs), we find that the approximation breaks down for some of the individual remnants like the Vela, but interestingly it still holds good for their combined total spectrum at the Earth. Moreover, we also find that the results obtained under this simple approximation are quite different from those calculated under an energy-dependent escape model which is favoured by diffusive shock acceleration models inside SNRs. Our study suggests that if SNRs are the main sources of CRs in our Galaxy, then the commonly adopted point-source model (with an energy-independent escape scenario) appears flawed for CR studies from the nearby SNRs.

Key words: cosmic rays – ISM: supernova remnants.

1 INTRODUCTION

Cosmic rays (CRs) with energies below the knee region (∼3 × 10^{15} \text{ eV}) are considered to be of galactic origin. Although the exact nature of their sources is not known, the most favourable candidates are the supernova remnants (SNRs). They are known to occur in our Galaxy at the rate of ∼1/30 yr^{-1}, with each explosion releasing a total kinetic energy of ∼10^{51} \text{ erg}. If approximately 10 per cent of this energy is converted into CRs, then the total power release is sufficient to maintain the CR energy density in our Galaxy which is measured to be around 1 \text{ eV cm}^{-3}.

It is also now theoretically established that SNR shock waves can accelerate CRs up to energies close to the knee by diffusive shock acceleration (DSA) mechanism (Bell 1978; Blandford & Eichler 1987). In a simple planar shocks model, such a mechanism naturally leads to a power-law spectrum of the form $E^{-\Gamma}$ with the exponent $\Gamma = 2$ for strong shocks. This value is found to be in good agreement with the radio observations of SNRs (Green 2009). In addition, direct evidence for the presence of high-energy particles up to few TeVs (1 TeV = 10^{12} \text{ eV}) inside SNRs comes from the detections of non-thermal X-rays (see e.g. Bamba et al. 2006; Parizot et al. 2006) and high-energy TeV $\gamma$-rays from several SNRs (e.g. Aharonian et al. 2006, 2008a,b,d; Albert et al. 2007). The non-thermal X-rays are best explained as synchrotron emission from high-energy electrons, while the origin of the TeV $\gamma$-rays is still not certain between the leptonic (via inverse Compton process) and the hadronic scenarios (via $\pi^0$ decays). If the high-energy $\gamma$-rays are of hadronic origin as indicated by the recent observations of several SNRs by the Fermi experiment (Abdo et al. 2009, 2010a,b,c), then the measured $\gamma$-rays can provide direct information about the spectral shape of the primary particles. But TeV measurements made by the new generation Cherenkov telescopes such as the HESS, MAGIC and VERITAS have found that many SNRs show $\Gamma$ ∼ (2.3–2.7) which is steeper than the expectations from DSA theory. The discrepancy becomes even more severe if we compare with the results of non-linear DSA theory which predicts a spectrum flatter than $\Gamma = 2$ (Caprioli, Amato & Blasi 2010, and references therein). Although this discrepancy is still not yet fully understood, for our study we will assume that SNRs are the main sources of CRs in our Galaxy.

Quite often, theoretical studies on the propagation of CRs assume the sources to be stationary and continuously distributed in the Galaxy. This simple assumption seems reasonable for calculating the Galactic average CR properties and for understanding the diffuse radiations produced by the interaction of CRs in the interstellar medium (ISM). But for CR studies in the vicinity of the sources where the influence of the source is expected to dominate over the background produced by the distant sources, the discrete nature of the sources (both in space and time) may become important. For instance, in the study of $\gamma$-ray emissions from the environment of the sources or from molecular clouds associated with them, the emission can be strongly dependent on the age and the distance of the source as discussed in Aharonian & Atoyan (1996), Gabri, Aharonian & Casanova (2009) and Casanova et al. (2010).

Similarly, for the study of CRs observed at the Earth, the uniform source distribution looks proper only for the distant sources but for the nearby sources, a more reasonable treatment would be to consider the discrete nature of the sources. For CR electrons at a few TeV energies, such treatment seems even more important because of...
their fast energy-loss rate. Electrons with energies greater than \(\sim 1\) TeV cannot travel distances more than \(\sim 1\) kpc in the Galaxy through diffusive propagation before they lose all their energies. Therefore, high-energy electrons from distant and old sources may not reach the Earth effectively, and it is possible that most of the TeV electrons that we observe are mostly produced by a few young nearby sources (Shen 1970; Atoyian, Aharonian \& V"olk 1995; Kobayashi et al. 2004; Delahaye et al. 2010, etc.). Also for the CR protons and other nuclear species, which do not suffer significant losses (the typical nuclear fragmentation-loss time-scale in our Galaxy is \(\sim 10^7\) yr) and for which we expect a strong background from the distant sources, the discrete treatment of the nearby sources can still be important especially at higher energies (Strong \& Moskalenko 2001; B"ulschung et al. 2005; Erlykin \& Wolfendale 2006, and references therein). This is because high-energy CRs diffuse relatively faster compared to the lower energy ones and hence they are expected to produce stronger fluctuations on their observable properties like the spectrum and the anisotropy (Thoudam 2008). Moreover, at these energies, the contribution from the recent sources may dominate and the effect of discreteness in time may also become important (Taitel et al. 2004).

In most of the studies mentioned above, the discrete sources are assumed to be point-like, thereby neglecting their finite size and the morphology. At first sight, the point-source approximation seems reasonable for sources whose size \(s \ll r\), the distance from the Earth. But for those whose size is comparable to the distance, the point-source approximation may break down and it looks more appropriate to take their size and morphology into account. Under the standard DSA theory, CRs are confined within the SNRs due to the strong magnetic turbulence generated by the CRs themselves and therefore it is reasonable to assume that CRs remain confined as long as the shocks remain strong enough to act as an efficient accelerator. For a typical interstellar matter density of \(1\) H cm\(^{-3}\), the confinement lasts until the SNRs age around \(10^3\) yr (Berezhko \& V"olk 2000). In reality, an energy-dependent confinement/escape scenario is expected (see e.g. Ptuskin \& Zirakashvili 2005; Caprioli, Blasi \& Amato 2009). Using the Sedov relation between the SNR age and the shock radius, if we assume an initial shock velocity of \(10^6\) cm s\(^{-1}\), we can roughly estimate that at the age of \(10^7\) yr, the remnant expands to a size of radius around 100 pc. Such a size is comparable to the distance of some of the nearest SNRs like the Geminga and the Loop1. The distance to the Geminga is estimated to be \(\sim 157\) pc (Caraveo et al. 1996) and that to the Loop1 is \(\sim 170\) pc (Egger \& Aschenbach 1995).

The argument just mentioned is purely based on the geometrical consideration, i.e. the source size compared to its distance, and we have not considered any possible effects due to the propagation of CRs in the Galaxy. It is now well accepted that CRs undergo diffusive propagation before they lose all their energies. Therefore, it is reasonable to assume that CRs remain confined to the strong magnetic turbulence generated by the CRs themselves. We assume the diffusion coefficient due to scattering by the magnetic field irregularities and the anisotropy (Thoudam 2008). Moreover, at these energies, especially at higher energies (Strong \& Moskalenko 2001; B"ulschung et al. 2005; Erlykin \& Wolfendale 2006, and references therein).

Recently, Ohira, Murase \& Yamazaki (2011) highlighted the importance of the finite-size source for the nearby SNRs considering the CR spectrum expected at the Earth. Although SNRs can have complex morphologies, that too different from each other, for simplicity we will consider a spherical geometry for our study. In one part, we will consider an energy-independent escape of CRs from the SNRs. This is discussed in Sections 2 and 3. In another part of our study, we will investigate the energy-dependent escape model under which CRs of different energies are assumed to escape at different times. This study is given in Section 4. Then in Section 5, we apply our study to the nearby known SNRs and compare the results obtained under the different source models. Finally in Section 6, we present an overview of our results and discuss their implications.

### 2 CR PROTON SPECTRUM FROM AN SNR

In the diffusion model, neglecting losses due to interactions in the ISM, the propagation of CR protons in the Galaxy can be described by (see e.g. Gaisser 1990, and references therein)

\[
\nabla \cdot (DN_p) + Q_p = \frac{\partial N_p}{\partial t},
\]

(1)

where \(N_p(r, E, t)\) is the differential proton density, \(E\) is the kinetic energy, \(D(E)\) is the diffusion coefficient and \(Q_p(r, E, t)\) is the source term, i.e. the proton production rate from the SNR. In equation (1), we also neglect other effects which are relevant mostly below a few GeVs like the convection due to the Galactic wind and the re-acceleration by the interstellar turbulence. We assume the diffusion coefficient to be spatially constant throughout the Galaxy and take \(D(E) = D_0(E/E_0)^3\) for \(E > E_0\), where \(D_0 = 2.9 \times 10^{28}\) cm\(^2\) s\(^{-1}\), \(E_0 = 3\) GeV and \(\delta = 0.6\) (Thoudam 2008). These values are different from those given by models based on diffusive re-acceleration. For instance, Trotta et al. (2011) gave a value of \(D_0 \sim 8.3 \times 10^{28}\) cm\(^2\) s\(^{-1}\) and \(\delta \sim 0.3\).

For sources within a distance of \(\sim 1\) kpc from the Earth which are also our main interest here, Thoudam (2007) showed that the CR spectrum is not much affected by the presence of the Galactic boundaries. In fact, \(D_0\) does depend on the boundaries and is proportional to the size of our Galactic halo (see e.g. Trotta et al. 2011). For our present study, we neglect such dependences and solve equation (1) without imposing any boundary conditions. We then obtain the well-known Green function \(G_p(r, r', t, t')\), i.e. the solution for a \(\delta\)-function source term \(Q_p(r, E, t) = \delta(r - r')\delta(t - t')\) as given below:

\[
G_p(r, r', t, t') = \frac{1}{8[\pi D(t - t')]^{3/2}} \exp \left\{ \frac{-(r - r')^2}{4D(t - t')} \right\}.
\]

(2)

The general solution of equation (1) can then be obtained using

\[
N_p(r, E, t) = \int_{-\infty}^{\infty} dr' \int_{-\infty}^{\infty} dt' G_p(r, r', t, t')Q_p(r', E, t').
\]

(3)

The source term in equation (3) can be written as

\[
Q_p(r', E, t') = q(r')q(E)dEq(t').
\]

(4)

where \(q(E)\) is the source spectrum, i.e. \(q(E)dE\) is the number of protons with energy between \(E\) and \(E + dE\) produced by the SNR. For this part of our study, we assume an energy-independent escape of CRs from the SNR. We will first consider the burst-like injection of particles followed later by the continuous injection case. Later on, in Section 4 we will discuss the energy-dependent escape model.
If we assume that the burst-like emission of particles occur at time \( t_0 \), we can write the temporal source term as \( q(t') = \delta(t' - t_0) \). Then, using equations (2) and (4) in equation (3), we get

\[
N_p(r, E, t) = \frac{q(E)}{8[\pi D(t - t_0)]^{3/2}} \int_{-\infty}^{+\infty} dr' \exp \left[ \frac{-(r^2 + r'^2)}{4D(t - t_0)} \right] q(r').
\]

(5)

The proton intensity can then be calculated using the relation \( I_p(E, t) \approx (c/4\pi)N_p(E, t) \), where \( c \) is the velocity of light. To make our calculations simpler, hereafter we take \( r = 0 \), i.e. we set the origin of the coordinate system at the position of the Earth.

### 2.1 Point-source approximation

If we assume the SNR to be a point source located at a distance \( r_s \) from the Earth, we can write

\[
q(r') = \delta(r' - r_s).
\]

(6)

Then, the proton density is obtained from equation (5) as

\[
N_p(E, t) = \frac{q_p(E)}{8[\pi D(t - t_0)]^{3/2}} \exp \left[ \frac{-r_s^2}{4D(t - t_0)} \right].
\]

(7)

Equation (7) represents the most commonly adopted solution for the CR spectrum from a nearby single source. For high-energy particles for which the diffusion radius defined as \( r_{\text{diff}} = \sqrt{4D(t - t_0)} \) is much larger than the distance to the SNR \( r_s \), the exponential term in equation (7) tends to 1 which implies

\[
N_p(E, t) \rightarrow \frac{q_p(E)}{8[\pi D(t - t_0)]^{3/2}}.
\]

(8)

For a power-law source spectrum given by \( q_p(E) = k_p E^{-\gamma} \) and for \( D(E) \propto E^\delta \), equation (8) shows that the spectrum of high-energy protons reaching us follows \( N_p(E) \propto E^{-(\gamma + {\delta/2})} \). Particles with \( r_{\text{diff}} > r_s \) are those which have already passed the Earth. Those with \( r_{\text{diff}} < r_s \) are the ones which have not yet reached effectively due to their slower diffusion and, therefore, their intensity is comparatively much suppressed.

### 2.2 The spherical solid source

Most of the SNRs are observed to roughly follow a spherical geometry and they come under three main categories: shell type, plerion type and composite type. Shell-type SNRs show a bright shell structure and they come under three main categories: shell type, plerion type and composite type. Shell-type SNRs and it is probably more relevant than the solid source model considered here closely represents the shell-type SNRs in radio as well as in X-rays is observed to peak near the surface which expands into the ISM with velocities of \( \sim (3 - 10) \times 10^3 \) cm s\(^{-1} \) (e.g. Cassiopeia A). Plerions also known as pulsar wind nebulae have a filled centre normally a pulsar powering high-energy particles into the ISM (e.g. Crab nebula). They do not show any shell-like features. Composite SNRs have both the shell structure and filled centre (e.g. IC443). The surface brightness of shell-type SNRs have a filled centre normally a pulsar powering high-energy particles responsible for the radio and the X-ray emissions also follow a similar distribution within the remnant.

Let us now consider a spherical solid source. We believe that this source model roughly represents the plerions and the composite-type SNRs. For this model, if \( r_0 \) denotes the position of the centre of the SNR from the Earth and \( r_0 \) represents the position of the source protons with respect to the SNR centre, we can write \( r' \) in equation (5) as \( r' = r_s + r_0 \) and then rewrite equation (5) as

\[
N_p(E, t) = \frac{q(E)}{8[\pi D(t - t_0)]^{3/2}} \int dr_0 \exp \left[ \frac{-(r_s + r_0)^2}{4D(t - t_0)} \right] q(r_0).
\]

(9)

In equation (9), the integral over the volume element in spherical geometry is given by

\[
\int dr_0 = \int_0^R r_0^2 \sin\theta_0 d\theta_0 \int_0^{2\pi} d\phi_0,
\]

where \( R \) denotes the radius of the SNR. We take the source spectrum in this case as \( q(E) = q_p(E)/V \), where \( q_p(E) \) is the source spectrum we took in the case of the point-source approximation (Section 2.1) and \( V = \frac{4}{3}\pi R^3 \) represents the total SNR volume. We assume that CRs are uniformly distributed throughout the SNR volume before releasing into the ISM, and we take the spatial source term as

\[
q(r_0) = \begin{cases} 1, & \text{for } r_0 \leq R \\ 0, & \text{otherwise} \end{cases}.
\]

(10)

Integrating equation (9) over \( \theta_0 \) and \( \phi_0 \), we get

\[
N_p(E, t) = \frac{q_p(E)}{r_s V \pi D(t - t_0)} \exp \left[ \frac{-r_s^2}{4D(t - t_0)} \right] \times \int_0^R r_0 \exp \left[ \frac{-r_0^2}{4D(t - t_0)} \right] \sinh \left( \frac{r_s r_0}{2D(t - t_0)} \right) dr_0.
\]

(11)

Using the properties \( \sinh(x) \approx x \) and \( e^x \rightarrow 1 \) for very small \( x \), it is easy to check that for very small \( R \), equation (11) reduces to the point-source solution (equation 7) at all energies.

### 2.3 The spherical surface source

If we assume that the CRs are distributed uniformly only on the surface of the SNR before they are released, the spatial source term in equation (9) can be written as

\[
q(r_0) = \delta(r_0 - R)
\]

(12)

and the source spectrum as \( q(E) = q_p(E)/A \), where \( A = 4\pi R^2 \) denotes the total surface area of the SNR. The CR density in this case is then obtained as

\[
N_p(E, t) = \frac{q_p(E) R}{r_s A \pi D(t - t_0)} \exp \left[ \frac{(R^2 + r_s^2)}{4D(t - t_0)} \right] \times \sinh \left( \frac{r_s R}{2D(t - t_0)} \right).
\]

(13)

Here again, we can note that equation (13) tends towards the point-source solution for very small values of \( R \). The spherical surface source model considered here closely represents the shell-type SNRs and it is probably more relevant than the solid source model for CR studies in our Galaxy. It is because according to the recent catalogue of Galactic SNRs, 78 per cent of the total known SNRs are of shell type, while the remaining 12 and 4 per cent are of composite and plerion types, respectively (Green 2009).

In Fig. 1 (top panel), we compare the spectra obtained under the different source models for \( t = 10^6 \) and \( 10^4 \) yr. The calculation assumes \( t_0 = 0 \), the SNR distance as \( r_s = 0.15 \) kpc and the diffusion constant as \( D_0 = 2.9 \times 10^{28} \) cm\(^2\) s\(^{-1}\). For our present illustration, we take the source spectral index as \( \Gamma = 2 \), which is the value predicted by DSA theories inside SNRs. Later on, in Section 5 when we apply our study to the nearby known SNRs, we use values which are determined based on the observed CR data. In Fig. 1 (top panel), we can see that for a given value of \( t \), the point-source solution (solid line) above some energy \( E_{\text{pt}} \) agrees well with the results of the surface source (dashed line) and the solid source (dotted line) models, while below \( E_{\text{pt}} \) the results are quite different. \( E_{\text{pt}} \) is roughly the energy at which \( r_{\text{diff}} = r_s \). We can check that for \( E > E_{\text{pt}} \) for which \( r_{\text{diff}} > r_s \), equations (11) and (13) tend towards

© 2011 The Authors, MNRAS 419, 624–637

larger values. In order to understand the effect of the source distance, equation (8) which is the asymptotic solution of the point-source approximation at high energies. The bottom panel shows the results for a larger value of the diffusion constant $D_0 = 2.9 \times 10^{29}$ cm$^2$ s$^{-1}$. The only difference between the two sets of results is that $E_{pt}$ is shifted towards lower values as $D_0$ increases. This shows that the point source becomes valid over a broader energy range as $D_0$ takes larger values. In order to understand the effect of the source distance, we show in Fig. 2 the results obtained for $r_s = 0.3$ kpc by keeping all other parameters the same as that in Fig. 1. On comparing the results in Fig. 2 to those in Fig. 1, we can see that apart from the scaling down of the flux and the right shifting of $E_{pt}$ due to the increased source distance, the differences between the different source models also become smaller. This is simply the geometrical effect mentioned in Section 1, i.e. as the source distance increases, the point-source approximation becomes more valid.

These results can be understood as follows. The diffusion radius $r_{\text{diff}} = \sqrt{D_0(t - t_0)}$, which is the effective distance from the SNR travelled by CRs due to diffusive propagation, is a strong function of $D_0$ and $t$. The larger the values of $D_0$ and/or $t$, the larger is $r_{\text{diff}}$ and the energy $E_{pt}$ which satisfy the condition $r_{\text{diff}} = r_s$ decreases. Similarly, for larger source distances $r_s$, we can understand that $E_{pt}$ shifts towards higher values. These results show that the point source can remain a valid approximation even for the nearby sources as long as the particles satisfy the condition $r_{\text{diff}} \gg (R, r_s)$. For $r_s = 0.15$ kpc, $D_0 = 2.9 \times 10^{29}$ cm$^2$ s$^{-1}$ and $t_0 = 0$ (Fig. 1, top panel),

we obtain $E_{pt} = 1.2 \times 10^5$ and 57 GeV for $t = 10^3$ and $10^4$ yr, respectively. The corresponding values for $r_s = 0.3$ kpc (Fig. 2, top panel) are found to be $1.2 \times 10^5$ and 575 GeV, respectively. For a given age or distance, the closer the older the source is, the lower is the $E_{pt}$. This is shown in Fig. 3 for $E_{pt} = 3$ GeV (solid line) and 10 GeV (dashed line). The area below each line represents the parameters space in $(r_s, t)$, where the point-source approximation works as a good approximation for all energies above the given
\(E_{\text{pc}}\). In the same figure, the black dots represent the nearby known SNRs with distances \(<1\) kpc from the Earth. References: (1) Blair, Sankrit & Raymond 2005; (2) Tatematsu et al. 1990; (3) Leahy & Aschenbach 1996; (4) Leahy & Tian 2007; (5) Braun, Goss & Lyne 1989; (6) Caraveo et al. 2001; (7) Miceli, Bocchino & Reale 2008; (8) Jian-Wen, Xi-Zhen & Jin-Lin 2005; (9) Strom 1994; (10) Plucinsky et al. 1996; (11) Caraveo et al. 1996; (12) Egger & Aschenbach 1995; (13) Katsuda, Tsunemi & Mori 2008 (where for the age we take the mean of 2700 and 4300 yr reported in the literature).

<table>
<thead>
<tr>
<th>SNR</th>
<th>Distance (kpc)</th>
<th>Age (yr)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cygnus Loop</td>
<td>0.540</td>
<td>1.0 × 10^4</td>
<td>1</td>
</tr>
<tr>
<td>HB21</td>
<td>0.800</td>
<td>1.9 × 10^4</td>
<td>2, 3</td>
</tr>
<tr>
<td>HB9</td>
<td>0.800</td>
<td>6.6 × 10^3</td>
<td>4</td>
</tr>
<tr>
<td>S147</td>
<td>0.800</td>
<td>4.6 × 10^3</td>
<td>5</td>
</tr>
<tr>
<td>Vela</td>
<td>0.294</td>
<td>1.12 × 10^4</td>
<td>6, 7</td>
</tr>
<tr>
<td>G299.2−2.9</td>
<td>0.500</td>
<td>5.0 × 10^3</td>
<td>8</td>
</tr>
<tr>
<td>SN185</td>
<td>0.950</td>
<td>1.8 × 10^4</td>
<td>9</td>
</tr>
<tr>
<td>Monogem</td>
<td>0.300</td>
<td>8.6 × 10^4</td>
<td>10</td>
</tr>
<tr>
<td>Geminga</td>
<td>0.157</td>
<td>3.4 × 10^4</td>
<td>11</td>
</tr>
<tr>
<td>Loop1</td>
<td>0.170</td>
<td>2.0 × 10^4</td>
<td>12</td>
</tr>
<tr>
<td>G114.3+0.3</td>
<td>0.700</td>
<td>4.1 × 10^4</td>
<td>8</td>
</tr>
<tr>
<td>Vela Junior</td>
<td>0.750</td>
<td>3.5 × 10^3</td>
<td>13</td>
</tr>
</tbody>
</table>

For \(t < T\), \(t_f = t\), which implies \(x_2 = \infty\). Then, using the property of the error function, \(\text{erf}(\sqrt{x_2}) = 1\) for \(x_2 = \infty\), equation (15) in this case becomes

\[
N_p(E, t) = \frac{q_s(E)}{4\pi r_s D T} \left[ 1 - \text{erf}\left(\sqrt{x_2}\right) \right].
\]

(16)

For high-energy particles for which the diffusion radius \(r_{\text{diff}} \gg r_s\), \(x_1 \to 0\), and because \(\text{erf}(\sqrt{x_2}) \to 0\) for \(x_1 \to 0\), the spectrum given by equation (16) follows a power law of the form \(N_p(E) \propto E^{-(t + \xi)}\), which is flatter than the spectrum we obtain in the burst-like injection model. A detailed discussion on this topic can also be found in Aharonian & Atoyan (1996) in the study of the CR spectrum in the vicinity of the sources.

For \(t > T\), \(t_f = T\) and \(x_2 = r_s^2/4D(T - t)\). For particles with large \(r_{\text{diff}}\) for which \(x_1 \to 0\), we can safely write \(x_2 \ll 1\) as \((t - T) < t\). Then, using the property \(\text{erf}(\sqrt{x_2}) \approx 2\sqrt{x_2}/\pi\) for \(x_2 \ll 1\), the particle spectrum (equation 15) in this case reduces to

\[
N_p(E, t) \approx \frac{q_s(E)}{4(\pi D)^{3/2} T \sqrt{T - t}}.
\]

(17)

The spectral shape of equation (17) follows \(N_p(E) \propto E^{-(t + \xi)}\) which is the same as in the case of the burst-like injection of particles discussed earlier (equation 8).

Similarly, using equation (14), we also obtain our results for the case of the solid and the surface source models by taking into account their proper source terms given by equations (10) and (12), respectively. The results are plotted in Fig. 4 (bottom panel) for

Figure 4. CR proton spectra for different source models from an SNR at \(r = 0.2\) kpc at different times \(t = (10^2, 10^3, 10^6)\) yr. Top: burst-like injection. Bottom: continuous injection with \(T = 10^3\) yr. Other model parameters are the same as in Fig. 1 (top panel).
$t = (10^2, 10^4$ and $10^6)$ yr along with the results obtained under the burst-like injection model (top panel) for comparison. The calculations in Fig. 4 assume $r_s = 0.2$ kpc, $D_0 = 2.9 \times 10^{23}$ cm$^{-3}$, $\Gamma = 2.0$ and $T = 10^9$ yr. We can see that at all times, the effect of assuming different source models are similar in both the types of injection. As discussed above, we can also see that for $t < T$ the spectra in the case of continuous injection are flatter than those in the burst-like injection case, while at $t \gg T$ they exactly follow the same behaviour as shown by the results at $10^9$ yr.

A short conclusion that we can draw at this stage of our study is that for very old sources ($t \gg T$), the effect of choosing different source geometry or a different particle injection model is negligible on the CR spectrum. Therefore, the widely adopted burst-like point-source approximation remains a good approximation for very old nearby sources at all the energies. However, for young nearby sources, the spectrum at high energies strongly depends on the type of the particle injection model, and at lower energies it starts depending on the physical size and the geometry of the source irrespective of the type of the injection model unless the source is really close to the Earth, i.e. only a few pc away as shown in Fig. 3.

3 HIGH-ENERGY ELECTRON SPECTRUM FROM AN SNR

The diffusive propagation of high-energy electrons in the Galaxy can be described by the following transport equation:

$$\nabla \cdot (D \nabla N_e) + \frac{\partial}{\partial t} [b(E)N_e] + Q_e = \frac{\partial N_e}{\partial t},$$  \hspace{1cm} (18)

where $N_e(E, t)$ is the density of electrons with the kinetic energy $E$, $b(E)$ is the energy-loss rate and $Q_e(r, E, t)$ denotes the electron injection rate into the ISM. The Green function of equation (18) can be obtained as given below (see e.g. Ginzburg & Syrovatskii 1964; Gratton 1972)

$$G_e(r, r', E, E', t, t') = \frac{1}{8 \pi f(E, E')} \exp \left[ \frac{-(r'-r)^2}{4f(E, E')} \right] \cdot \delta[t' - t + g(E, E')],$$  \hspace{1cm} (19)

where

$$f(E, E') = \int_{E}^{E'} \frac{D(u)}{b(u)} du \quad \text{and} \quad g(E, E') = \int_{E}^{E'} \frac{1}{b(u)} du.$$

For our present study, we assume that the energy loss of the electrons is due to synchrotron and inverse Compton interactions which are true mostly for energies $\geq 10$ GeV. We take

$$b(E) = a E^2,$$  \hspace{1cm} (20)

where $a = 1.0 \times 10^{-16} (w_{\text{ph}} + w_B)$ GeV s$^{-1}$ and $w_{\text{ph}}$ and $w_B$ represent the energy densities in eV cm$^{-3}$ for the background photons and the magnetic field, respectively. Equation (20) assumes that the inverse Compton scattering of the background photons occurs in the Thompson regime.

The general solution of equation (18) is given by

$$N_e(r, E, t) = \int_{-\infty}^{\infty} dr' \int_{-\infty}^{\infty} dE' \int_{-\infty}^{\infty} E' \int_{-\infty}^{\infty} dE G_e(r, r', E, E', t, t') \times Q_e(r', E', t').$$  \hspace{1cm} (21)

For an energy-independent burst-like injection of electrons at time $t_0$, we take the source term as $Q_e(r', E', t') = q(r')g(E')\delta(t' - t_0)$, where $q(E) \propto E^{-\Gamma}$ denotes the source spectrum. Now, setting $r = 0$ as we did for the protons in Section 2 and performing the integrals over $E'$ and $t'$, equation (21) becomes

$$N_e(E, t) = \frac{q(E)}{8\pi C^{3/2}} \left(1 - \frac{E}{E_i}\right)^{\Gamma - 2} \int_{-\infty}^{\infty} dr' \times \exp \left[ -\frac{r'^2}{4C} \right] g(r'),$$  \hspace{1cm} (22)

where $E_i = E/(a(t - t_0))$ is the energy at which the energy-loss time is equal to $(t - t_0)$,

$$C = \frac{D(E)}{a(1-\delta)E} \left[1 - \left(1 - \frac{E}{E_i}\right)^{-1-\delta} \right]$$  \hspace{1cm} (23)

and $\delta$ is the index of the diffusion coefficient. Equation (22) is valid for electrons with energies $E < E_i$. For $E > E_i$, $N_e = 0$.

3.1 Point-source approximation

For a point source described in equation (6) located at a distance $r_s$, the electron spectrum at time $t$ can be obtained from equation (22) as given below

$$N_e(E, t) = \frac{q_e(E)}{8\pi C^{3/2}} \left(1 - \frac{E}{E_i}\right)^{\Gamma - 2} \exp \left[ -\frac{r_s^2}{4C} \right],$$  \hspace{1cm} (24)

where $q_e(E) = k_e E^{-\Gamma}$ is the source spectrum for the point source. From equation (23), we can see that in the energy region $E \ll E_i$ where the effect of the energy loss is less important, $C \approx D(t - t_0)$ and equation (24) tends towards the point-source solution for CR protons (equation 7). Therefore, high-energy electrons whose $r_{\text{diff}} \gg r_s$ and $E \ll E_i$ have spectrum which follows $N_e(E) \propto E^{-(\Gamma + 1/2)}$, which is similar to the asymptotic solution of high-energy protons (equation 8).

3.2 The spherical solid source

Following exactly the same procedure as for the protons described in the previous section, we obtain the electron spectrum for the spherical solid source as

$$N_e(r, E, t) = \frac{q_e(E)}{r_s V \sqrt{\pi} C} \left(1 - \frac{E}{E_i}\right)^{\Gamma - 2} \exp \left[ -\frac{r_s^2}{4C} \right] \times \int_0^R r_0 \exp \left( -\frac{r_0^2}{4C} \right) \sinh \left( \frac{r_0 R}{2C} \right) dr_0.$$  \hspace{1cm} (25)

3.3 The spherical surface source

We also obtain the solution for the spherical surface source as given below

$$N_e(r, E, t) = \frac{q_e(E) R}{r_s A \sqrt{\pi} C} \left(1 - \frac{E}{E_i}\right)^{\Gamma - 2} \exp \left[ -\frac{(R^2 + r_s^2)}{4C} \right] \times \sinh \left( \frac{r_s R}{2C} \right).$$  \hspace{1cm} (26)

It is easy to check that for very small values of $R$, the solutions for the solid-source (equation 25) and the surface-source (equation 26) models reduce to the point-source solution (equation 24).

The solutions we have obtained above are based on the burst-like injection of electrons from the SNR. For the case of continuous
injection, the solutions are given by

\[ N_s(E,t) = \int_0^{t_f} dt' \frac{q_s(E)}{8(\pi C)^{3/2}} \left(1 - \frac{E}{E_i}\right)^{\Gamma-2} \times \int_{-\infty}^{\infty} dr' \exp \left(-\frac{r'^2}{4C}\right) q(r'), \tag{27}\]

where \( t_f \) and \( q_s(E) \) bear the same definitions as defined in the case of protons and \( E_i \) replaced by \( E'_i \). In this case, we can check that for energies \( E \ll E'_i \), \( C \rightarrow D(t-t') \) and equation (28) reduces to a solution similar to that of the CR protons (equation 15).

Therefore, the same discussions we presented in the previous section for the protons under the continuous injection model also apply to the electrons. At time \( t < T \), the spectra of high-energy electrons with \( E \ll E'_i \) and whose diffusion radii \( r_{\text{diff}} \gg r_s \) follow \( N_s(E) \propto E^{-(\Gamma+4)} \) and at \( t > T \), they follow \( N_s(E) \propto E^{-(\Gamma+\frac{3}{2})} \) which is similar to the result obtained in the burst-like injection model (Section 3.1). More discussions on the different types of electron spectra generated by a CR source under different particle injection models can also be found in Atoyan et al. (1995).

Using equation (27), we also calculate the spectra for the solid and the surface sources under the continuous injection model. The results are plotted in Fig. 5 (bottom panel) for \( t = (10^2, 10^4, 10^6) \) yr. The top panel shows the results for the case of burst-like injection. The calculations assume \( \Gamma = 2.0, \delta = 0.6, D_0 = 2.9 \times 10^{28} \) cm\(^2\) s\(^{-1}\), \( r_s = 0.2 \) kpc, \( T = 10^5 \) yr and \( n_0 = 0 \). The magnetic field in the ISM is taken as 6 \( \mu \)G (Beck 2001). The total energy density of the background photon field is assumed to be \( w_{\text{ph}} = w_{\text{MBR}} + w_{\text{op}} \), where \( w_{\text{MBR}} = 0.25 \) eV cm\(^{-3}\) is the energy density of the microwave background and \( w_{\text{op}} = 0.6 \) eV cm\(^{-3}\) that of the ultraviolet–near-infrared–optical radiation field. The latter is taken from the estimates given in Shibata, Ishikawa & Sekiguchi (2011) for the galactic-centric distance of 8.5 kpc, which they obtain using the data provided by GALPROP (Porter et al. 2008). On comparing top and bottom panels of Fig. 5, we can note that at \( t < T \), apart from the difference in the slope of the spectra, there are sharp spectral breaks present in the case of burst-like injection. These are due to the effect of fast the energy-loss rate for high-energy electrons. Electrons with energy \( E > E_i \) are lost before reaching the Earth. We also note that the differences between the different source models below \( E_{\text{th}} \) are similar in both the types of the injection model. At very late times \( t \gg T \), the spectra become independent of the injection model or of the source model and they exhibit the same shapes with breaks at \( E = E_i \).

On comparing the results of electrons (Fig. 5) to those of the protons (Fig. 4), we can see that except for the presence of spectral breaks in the case of electrons, the results are quite similar in all other respects. Even the differences between the results obtained for different source models are similar. Therefore, the overall conclusions on the validity of the point-source approximation that we had drawn earlier for the protons also apply to the electrons.

\[ E_{\text{esc}} \propto B_s R_s u_s. \tag{29}\]

There are strong theoretical arguments which suggest that CRs might amplify the magnetic fields near the shock surface (see e.g.
Caprioli et al. 2009). This idea is also supported experimentally by the recent observations of thin X-ray filaments inside several SNRs, which are most likely synchrotron emissions of high-energy electrons in the presence of strong magnetic fields of the order of $\sim(100–1000)$ $\mu$G (Völk, Berezhko & Ksenofontov 2005). Taking such possible amplification into account, we can assume that the magnetic field scales with the shock velocity as $B_s \propto u_s^\alpha$, with the index $\alpha$ representing the degree of amplification. Some studies suggest that $\alpha$ can reach values as high as 1.5 (Bell 2004).

One reasonable assumption of DSA theory is that CRs do not escape during the free expansion phase of the SNR evolution. It is because shock waves travelling at some constant velocity can always overtake particles undergoing diffusive motion (Drury 2011). However, during the Sedov phase when the shock velocity decreases with the age $t$ as $u_s \propto t^{-0.6}$ and the shock radius increases as $R_s \propto t^{0.4}$, some of the high-energy CRs can start escaping because of their relatively larger diffusion length ($t_{\text{diff}} > t_{\text{esc}}$). Therefore, under the Sedov scaling, the escape energy at any stage during the evolution can be obtained using equation (29) as

$$E_{\text{esc}} \propto t^{-(0.2+0.6d)}.$$  \hfill (30)

This gives

$$E_{\text{esc}} \propto \begin{cases} t^{−0.2}, & \text{for } d = 0 \\ t^{−1.1}, & \text{for } d = 1.5 \end{cases}.$$  \hfill (31)

In deriving equation (31), we assume that $D_s(E)$ scales linearly with $E$. But the exact dependence is still not well understood and depends on some poorly known yet important quantities like the spectral distribution of the self-excited turbulence waves, their dissipation rate and their CR scattering efficiencies. Moreover, magnetic field amplification and the dynamical reaction of the accelerated particles on the shock structure are also not fully understood. Due to these uncertainties, a simple but reasonable approach which is commonly followed is to parametrize the escape energy as given below (Gabici et al. 2009; Ohira et al. 2011):

$$E_{\text{esc}} = E_{\text{max}} \left( \frac{t}{t_{\text{sed}}} \right)^{-\alpha}.$$  \hfill (32)

where $E_{\text{max}}$ is the maximum CR energy and $t_{\text{sed}}$ denotes the start of the Sedov phase. We assume $E_{\text{max}} = 10^6$ GeV ($=1$ PeV) and $t_{\text{sed}} = 500$ yr for our study. Equation (32) assumes that the escape of the highest energy particles starts at the onset of the Sedov phase itself. For detailed studies of particle escape from SNRs, see e.g. Ptuskin & Zirakashvili (2005) and Caprioli et al. (2009, 2010). Using equation (32), we can calculate the escape time $t_{\text{esc}}$ as a function of energy as

$$t_{\text{esc}}(E) = t_{\text{sed}} \left( \frac{E}{E_{\text{max}}} \right)^{-1/\alpha}.$$  \hfill (33)

At some later stage of the SNR evolution when the shock slows down and does not efficiently accelerate the CRs, the turbulence level in the vicinity of the shock goes down and no particles can remain confined effectively within the remnant. At this stage, we can assume that all the CR escape into the ISM. As previously mentioned, for an ISM density of $1$ H cm$^{-3}$, this happens at around $10^5$ yr after the supernova explosion (Berezhko & Völk 2000). Taking this into account, the CR escape time for our study is taken as

$$T_{\text{esc}}(E) = \min \{ t_{\text{esc}}(E), 10^5 \text{yr} \}.$$  \hfill (34)

Using the Sedov relation between the shock radius and the SNR age, we can also calculate the escape radius $R_{\text{esc}}$ which we define as the radius of the SNR at the time when CRs of energy $E$ escape as follows:

$$R_{\text{esc}}(E) = 2.5v_0 t_{\text{sed}} \left[ \frac{T_{\text{esc}}^{0.4}}{t_{\text{sed}}} \right]^{0.6}.$$  \hfill (35)

In equation (35), $v_0$ represents the initial shock velocity, i.e. the velocity at $t = t_{\text{sed}}$ which we take as $10^6$ cm s$^{-1}$ for our study.

Equation (34) is plotted in Fig. 6 (top panel) where different lines correspond to different values of $\alpha$: solid (0.2), dashed (1.1) and dotted (2.0). The plots show that even for the fixed values of $E_{\text{max}}$ and $t_{\text{sed}}$, the energy dependence of $T_{\text{esc}}(E)$ strongly depends on the value of $\alpha$. For $\alpha = 0.2$, except for particles with energies greater than $3.5 \times 10^5$ GeV all the particles remain confined till the end of the SNR evolution. As the value of $\alpha$ increases, lower energy particles start escaping at relatively early stages. For $\alpha = 1.1$ and 2.0, only particles with energies up to $3 \times 10^5$ and 25 GeV, respectively, are confined till the end of the evolution. The bottom panel shows the corresponding values of $R_{\text{esc}}(E)$ calculated using equation (35). For the assumed value of $v_0$, the CR escape starts when the remnant expands to a radius of $\sim 5$ pc and continues until it expands up to $\sim 100$ pc. The latter value denotes the maximum CR confinement radius in our study.

For our calculations in the following, we will assume that at the time of escape from the SNRs, CRs are distributed uniformly at the shock surface. This assumption is similar to that of the spherical surface source discussed in Sections 2 and 3.

© 2011 The Authors, MNRAS 419, 624–637
energies (equation 8). An additional effect of large evolution reduces to that of the point-source approximation at high.

4.1 CR proton spectrum from an SNR

For CR protons, the source term components in the energy-dependent escape model can be written as

\[ q(r_0) = \delta(r_0 - R_{\text{esc}}), \]
\[ q(E) = \frac{q_p(E)}{A_{\text{esc}}}, \]
\[ q(t') = \delta(t' - T_{\text{esc}}), \]

where \( q_p(E) \) is the point-source spectrum given in Section 2.1, \( T_{\text{esc}} \) and \( R_{\text{esc}} \) are given by equations (34) and (35), respectively, and \( A_{\text{esc}} = 4\pi R_{\text{esc}}^2 \). Now, the proton spectrum in this case is obtained using equation (13) by substituting the above source parameters as

\[ N_p(E, t) = \frac{q_p(E) R_{\text{esc}}}{r_0 A_{\text{esc}} \sqrt{\pi D(t - T_{\text{esc}})}} \exp \left[ -\frac{(R_{\text{esc}}^2 + r_0^2)}{4D(t - T_{\text{esc}})} \right] \times \sinh \left( \frac{r_0 R_{\text{esc}}}{2D(t - T_{\text{esc}})} \right). \]

The expected spectrum is almost independent of the chosen values of \( \alpha \), \( T_{\text{esc}} \) and \( R_{\text{esc}} \). This is more clearly visible in the results obtained for \( t = 10^8 \) yr, where the two spectra are almost identical to each other over the energy range considered here.

Although taking different values of \( \alpha \) results into different types of spectrum especially at the lower energies at a given time, hereafter we will adopt \( \alpha = 2.0 \) for our study. The effects of choosing other values of \( \alpha \) on our results will be discussed later in Section 6.

4.2 High-energy electron spectrum from an SNR

To proceed, we recall equation (26), which represents the electron spectrum \( N_e(E) \) for the spherical surface source obtained under the energy-independent escape model. In that equation, electrons of energy \( E \) observed at time \( t \) had an initial energy \( E' \) at the time of their escape given by

\[ E' = \frac{E}{1 - aE(t - t_0)}, \]

where \( t_0 \) denotes the escape time from the SNR. We can reverse the situation and calculate the energy of an electron after time \( t \) for a given initial energy \( E' \) as

\[ E = \frac{E'}{1 + aE(t - t_0)}. \]

If \( q(E) \) represents the source spectrum of electrons with initial energy \( E' \) which escape at time \( T_{\text{esc}}(E') \), their energy \( E \) at time \( t \) is given by equation (40) and their number density is obtained using equation (26) as given below

\[ N_e(E, t) = \frac{q_e(E) R_{\text{esc}}'}{r_0 A_{\text{esc}}' \sqrt{\pi C' \delta}} \left[ 1 + aE(t - T_{\text{esc}}) \right]^2 \times \exp \left[ -\frac{(R_{\text{esc}}'^2 + r_0^2)}{4C'} \right] \sinh \left( \frac{r_0 R_{\text{esc}}'}{2C'} \right), \]

where \( C' = \frac{D(E')}{a(1 - \delta)E'} \left( 1 - \left[ 1 + aE(t - T_{\text{esc}}') \right]^{\frac{1}{\delta - 1}} \right) \) and \( R_{\text{esc}}' = R_{\text{esc}}(E') \), \( T_{\text{esc}}' = T_{\text{esc}}(E') \) and \( A_{\text{esc}}' = A_{\text{esc}}(E') \).

Using equation (41), we calculate the electron spectra at different times for a source distance \( r_0 = 0.2 \) kpc. The results are shown in Fig. 8. On comparing with the results obtained for the protons shown in Fig. 7 (\( \alpha = 2.0 \)), one can note that the major difference is the presence of additional breaks at higher energies which are due to the effect of radiative energy losses. The breaks at the lower energies, which are due to the effect of \( T_{\text{esc}} \), are seen at the same energies for both types of particles. The electron spectrum between the breaks follows a power-law \( \Gamma + \frac{\delta}{\delta - 1} \) similar to the proton spectrum, and at very late times (say at \( t = 10^8 \) yr), it also tends towards the point-source solution. These results show that also in the case of the energy-dependent escape scenario, for very old sources (\( t \gg 10^8 \) yr), the spectrum at all energies can be well approximated by the simple point-source solution.

5 APPLICATION TO THE NEARBY SNRS

In this section, we shall apply our study to the nearby known SNRs listed in Table 1 with distances < 1 kpc from the Earth. It should
be mentioned that some of the age and the distance parameters listed in Table 1 carry large uncertainties. For instance, the distance to the Geminga was measured to be 157 pc using Hubble Space Telescope (HST) observations (Caraveo et al. 1996), but recently, again using HST measurements, Faherty, Walter & Anderson (2007) reported the distance of Geminga to be 250 pc. For the Cygnus Loop, Minkowski (1958) reported a distance of 770 pc, whereas measurements based on HST observations claimed a distance of 440 pc (Blair et al. 1999). Recent measurements further claimed the distance to be 540 pc (Blair et al. 2005). For HB21, Tatematsu et al. (1990) measured a distance of 800 pc and Leahy & Aschenbach (1996) estimated an age of $1.9 \times 10^4$ yr, while later Byun et al. (2006) suggested a distance of 1.7 kpc and Lazendic & Slane (2006) estimated an age of $5.6 \times 10^3$ yr. Leahy & Aschenbach (1995) estimated the distance and age of HB9 as 1 kpc and $7.7 \times 10^3$ yr, respectively, and Leahy & Tian (2007) suggested a distance of 800 pc with the Sedov age of $6.6 \times 10^3$ yr and $(4-7) \times 10^3$ yr based on the evaporation cloud model. The lack of precise information on these parameters can affect our results because of the strong dependence of the CR spectra on these parameters.

For our study, we will assume that the proton source index $\Gamma = 2.13$ so that for $\delta = 0.6$, we get $\Gamma + \delta = 2.73$ the observed proton spectral index at the Earth (Haino et al. 2004). It should be noted that the value of the source index can depend on the choice of the propagation model and different propagation models may take different values. For instance, models based on diffusive re-acceleration in the Galaxy favour a diffusion index of $\delta \sim 0.3$ which corresponds to a source index of $\Gamma \sim 2.4$ (Trotta et al. 2011). This is steeper than the value adopted in our present work, which is based on a purely diffusive model of CR propagation. For the CR electrons, to get the source index, we first determine the background spectrum. This is done by fitting the observed data between 10 and 200 GeV provided by the Fermi and the PAMELA experiments (Ackermann et al. 2010b; Adriani et al. 2011). We assume that this is the energy region where the contamination due to the local sources as well as the effect of the solar modulation is minimum. From the fit, the background spectral index is found to be $3.10 \pm 0.01$. Under the diffusive propagation model, CR electrons produced by a uniform and stationary source distribution, and subject to radiative losses during their propagation in the Galaxy result into an equilibrium spectrum given by $E^{-\alpha}$ where $\beta = (1 - \delta)/2$ (see e.g. Thoudam & Hörandel 2011). Using the value of the background index obtained from the fit, we get the electron source index as $\Gamma = 2.3$. This is the value we will adopt for the rest of our calculations for the electrons. Furthermore, in the following we will assume that 10 per cent of the supernova explosion energy of $10^{50}$ erg converts into CR protons and 0.1 per cent into the electrons. All these parameters are assumed to be the same for all the SNRs.

### 5.1 CR protons

First, we compare the results of the point-source approximation with those of the spherical solid and the surface source models which are all based on the energy-independent escape model. These are shown in Fig. 9 where different lines represent different source models: thin solid (point), dashed (surface) and dotted (solid). The thick solid line represents the fitted observed proton spectrum given in Haino et al. (2004). The calculation assumes a burst-like injection of particles at $t_0 = 0$ and that each SNR produces $10^{50}$ erg of CR protons. Other model parameters are $\Gamma = 2.13$, $D_0 = 2.9 \times 10^{28}$ cm$^2$ s$^{-1}$, $\delta = 0.6$ and $E_0 = 3$ GeV.

from the fit, the electron source index as $\Gamma = 2.3$. This is the value we will adopt for the rest of our calculations for the electrons. Furthermore, in the following we will assume that 10 per cent of the supernova explosion energy of $10^{50}$ erg converts into CR protons and 0.1 per cent into the electrons. All these parameters are assumed to be the same for all the SNRs.

![Figure 8. CR electron spectra at different times under the energy-dependent escape model for $\alpha = 2.0$. Other model parameters are the same as those in Fig. 7.](image)

![Figure 9. CR proton spectra from nearby SNRs listed in Table 1 for the three different source models: point source (solid line), surface source (dashed line) and solid source (dotted line). The contributions from the individual SNRs are labelled by their names and their total contributions as 'total local SNRs'. The thick solid line represents the fitted total observed spectrum, $1.37 \times (E/\text{GeV})^{-2.73}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ GeV$^{-1}$ taken from Haino et al. (2004). Our calculation assumes an energy-independent burst-like injection of particles at $t_0 = 0$ and that each SNR produces $10^{50}$ erg of CR protons. Other model parameters are $\Gamma = 2.13$, $D_0 = 2.9 \times 10^{28}$ cm$^2$ s$^{-1}$, $\delta = 0.6$ and $E_0 = 3$ GeV.](image)
sharp cut-offs in the individual spectra due to radiative energy losses. For instance, the strong peak at $E \sim 10^4$ yr is due to the effect of SN185. From the figure, we can note that at energies greater than a few TeVs our results, which are based on a pure power-law source spectrum, significantly overpredict the data. Taking larger values of $t_0$ can suppress the contributions of Vela, G299.2–2.9 and SN185 which are the dominant contributors at high energies. For $t_0 = 2 \times 10^4$ yr, their contributions will be completely removed. This points towards the importance of source modelling in order to understand the contribution of local sources in the high-energy electron spectrum. One common way to handle this problem is to assume an exponential cut-off $\exp(-E/E_c)$ in the source spectrum at a few TeVs (see e.g. Delahaye et al. 2010). Another possibility is that the high-energy electrons might have suffered significant energy losses within the SNR itself before they are released into the ISM (Thoudam & Hörandel 2011). Therefore, electrons at higher energies might be released with a spectrum steeper than the lower energy ones. For the present study, we adopt the much simpler exponential cut-off, and in Fig. 12 we show the results obtained for $E_c = 2$ TeV. We can see that the shape of the total local spectrum in the TeV region is now determined mostly by the exponential cut-off and the irregular structures present near the highest energies in Fig. 11 no longer exist. In Fig. 12, the thick dashed line represents the background spectrum (which we obtain as mentioned before) with an index of 3.1 and an exponential cut-off at 2 TeV. The thick solid line represents the total background plus the nearby SNR contribution obtained in the point-source approximation. Detailed calculations of the background spectrum taking into account the various source models discussed here will be presented elsewhere.

For the energy-dependent escape model, the results are shown in Fig. 13 for a pure power-law source spectrum. In the figure, we also show for comparison the total local spectra obtained in the case of the point-source approximation (dashed line in Fig. 11). The total spectra show several irregular features and spikes. These features are stronger than the ones present in the proton spectrum which is due to the presence of additional breaks in the individual electron spectra at high energies. The position of these spikes not only depends on the age and distance of the individual SNRs, but also on the assumed energy-dependent escape model (i.e. on the parameters $\alpha$, $t_{\text{sed}}$ and $E_{\text{max}}$).

### 5.2 Electrons

The electron spectra for the point-, solid- and the surface-source models are shown in Fig. 11. In the figure, the data are from the Fermi (Ackermann et al. 2010b), PAMELA (Adriani et al. 2011) and the HESS (Aharonian et al. 2008c, 2009) experiments. As in the case of protons, the total electron spectra also do not show any differences between the different models. However, unlike in the case of protons, the total electron spectra show some irregular features near the highest energies which are due to the effects of

![Figure 11: Electron spectrum from the nearby SNRs listed in Table 1 for the three different source models: point source (solid line), surface source (dashed line) and solid source (dotted line). We assume a burst-like injection of particles and a pure power-law source spectrum of index $\Gamma = 2.3$ with each SNR producing 10$^{48}$ erg of CR electrons. All other model parameters remain the same as in Fig. 9. The data are taken from the Fermi, PAMELA and HESS experiments.](image1)

![Figure 12: Same as in Fig. 11 but for a source spectrum with an exponential cut-off at $E_c = 2$ TeV. The thick dashed line represents the background spectrum (see the text for details) and the thick solid line represents the total background plus nearby SNRs in the point-source approximation. The data are the same as in Fig. 11.](image2)
high energies, the point source still remains a good approximation. Under the energy-independent particle escape model, we found that the effects of the finite-source size are similar in both types of particle injection models considered in our study: the burst-like and the continuous injection. For very old nearby sources \((t \gtrsim 10^5\,\text{yr})\), we have found that the results are independent of both the source size and the particle injection model and hence the burst-like point-source model represents a good approximation at all energies. We have also shown in Fig. 3 that for a given value of the CR diffusion coefficient, there is a certain parameter space in \((r_s, t)\) under which the point-source approximation remains valid for CRs of our interest, i.e. with energies \(E > (3 - 10)\,\text{GeV}\). When applied to the nearby known SNRs within 1 kpc, interestingly we have found that their total spectra almost remain the same in the three different source models, although some of the individual SNRs like Vela show differences between the models (Figs 9 and 11). We found that it is because at low energies where the point-source approximation is most likely to break down, the local spectrum is dominated by the Monogem and the Loop1. These SNRs are quite old with the Monogem age \(\sim 8.6 \times 10^4\,\text{yr}\) and Loop1 \(\sim 2.0 \times 10^5\,\text{yr}\) due to which their CR spectra at the Earth are independent of their sizes and are well represented by the point-source solutions.

We have also studied an energy-dependent escape scenario where CRs of different energies are assumed to escape at different times during the SNR evolution. We assumed that the escape time follows, \(t_{\text{esc}} \propto E^{-1/\alpha}\) with \(\alpha\) chosen to be equal to 2.0. Under this model, we assumed that the highest energy particles escape the remnant at the start of the Sedov phase followed by the lower energy ones at later times. For \(E_{\text{max}} = 1\,\text{PeV}\), \(t_{\text{sed}} = 500\,\text{yr}\) and the maximum CR confinement time of \(10^5\,\text{yr}\) adopted for our study, we found \(t_{\text{esc}} = (500 - 10^5)\,\text{yr}\) and the escape radius \(R_{\text{esc}} = (5 - 100)\,\text{pc}\) for energies \(E = (1\,\text{PeV} - 25\,\text{GeV})\). For young sources, the spectrum obtained under this model shows breaks at lower energies which are due to the longer confinement times at those energies. At high energies, the results are very similar to those of the point-source approximation. This is not just because of the small values of \(R_{\text{esc}}\) at high energies, but also due to their large values of \(D(E)\) at these energies. In fact, we have shown in Section 2 that even for a large escape radius of 100 pc, the point source still represents a good approximation at high energies (see e.g. Fig. 1, surface source). Therefore, it should be understood that it is not the small \(R_{\text{esc}}\), but actually the large \(D(E)\) which is responsible for the point-source validity at high energies under the energy-dependent escape model. When applied to the nearby known SNRs, we have found that the results obtained under this model are significantly different from those obtained under the point-source approximation. The total local spectra show more irregular structures as compared to the point-source results. Also, we have noted that there is a big dip between around \(10^2\) and \(3 \times 10^3\,\text{GeV}\) which is mainly due to the low-energy cut-off in the Vela spectrum (Figs 10 and 13). These results seem to suggest that if SNRs are the main sources of CRs in our Galaxy, then the widely adopted point-source approximation with an energy-independent escape scenario appears flawed for CR studies from the nearby SNRs.

For the protons, the irregular spectral features that we have found in the energy-dependent escape model may be suppressed by the dominant background produced by distant sources and hence may not show up distinctly in the total observed spectrum. But for the electrons they can possibly show up to detectable levels especially at TeV energies where the background level is expected to be significantly less. Recently, Kawanaka et al. (2011) proposed that such spectral features can be used to estimate the CR confinement time.
inside the SNRs. Their study assumed a single nearby source having characteristics similar to that of the Vela remnant. It should be noted that the position and the strength of such features strongly depend on the CR escape model especially on the $\alpha$ parameter and also on $E_c$ (if there is an exponential cut-off in the source spectrum). For instance, assuming $E_c > 2$ TeV would produce stronger features and vice versa compared to our results shown in Fig. 14. Similarly, assuming $\alpha < 2.0$ would produce stronger peaks at comparatively higher energies as low-energy CRs would be confined for relatively longer times as indicated by Figs 6 and 7, and taking $\alpha > 2.0$ would smoothen the peaks as low-energy CRs would also start escaping at early times. For $\alpha \gg 2.0$, the energy-dependent results will tend towards the point-source results obtained for $t_0 = t_{sed}$. These can be understood from Fig. 15 where in the top panel we have shown the electron spectrum for the case of pure power-law source spectrum (which corresponds to $E_c = \infty$) for different values of $\alpha$: 1.8 (solid line), 2.0 (dashed line) and 2.5 (dotted line). We can clearly see the left shifting of the peak between 1 and 10 TeV as $\alpha$ increases from 1.8 to 2.5. In the bottom panel, we have shown the spectra calculated for the case of $\alpha = 1.8$ for source spectra with the exponential cut-offs at $E_c = 10$ TeV (solid line), 2 TeV (dashed line) and 1 TeV (dotted line). Here again, we can note that the peak at around 5 TeV grows stronger as $E_c$ takes larger values.

If we look into the HESS electron data, there is an indication of an abrupt rise at the highest measured energy. If future better sensitive experiments such as the CTA and the CALET provide good quality data at these energies, then that would indeed provide useful information to understand CR escapes from some of our nearby SNRs. However, the large uncertainties involved in the age and the distance estimates of some of these SNRs may be an issue because of the strong dependence of the CR spectrum on these parameters. Regarding this, measurements of electron anisotropies both amplitude as well as its direction at these energies might also be important in order to identify the dominant source.

Recently, Di Bernardo et al. (2011) studied the contributions of the nearby pulsars and the SNRs to the high-energy electron spectrum. One of their conclusions is that a strong contribution from the nearby SNRs is not supported by the recent upper limits on the electron anisotropies provided by the Fermi–Large Area Telescope (LAT) observations (Ackermann et al. 2010a). But it should be noted that their calculations assumed the sources to be burst-like point sources emitting CR particles independent of energy. In Fig. 14, we show that between $\sim 100$ GeV and 2 TeV, the contribution from the nearby SNRs is significantly larger in the point-source approximation than in the energy-dependent model. We believe that a more realistic treatment of the particle escape model from the SNRs may change their conclusion. Other class of sources which might also produce significant contributions to the high-energy leptonic (electron plus positron) spectrum are pulsars and dark matter. Models based on these sources are motivated mostly by the detection of the rise in the positron fraction above $\sim 10$ GeV by the PAMELA experiment (Adriani et al. 2009). If we assume that positrons are produced only during the interaction of the primary CRs with the interstellar gas, the positron fraction is expected to decrease with energy which is in contrast to the observations. A possible solution to this problem among others may be the presence of one or more nearby positron sources like pulsars or dark matter (see e.g. Grasso et al. 2009, and references therein). Future measurements of electron anisotropies with better sensitivities and also the absolute positron spectrum at high energies can provide better understanding of the nature and the type of the dominant source(s).

Moreover, a good understanding of the background contribution would also be crucial. In an earlier paper, we had presented calculations of the averaged background based on a simple energy-independent model of CR confinement within the SNRs (Thoudam & Hörandel 2011). In future, we will present background estimates for both the protons and the electrons, taking into account the energy-dependent confinement/escape of particles. The calculation will include the various energy-loss and interaction processes taking place during the time particles are confined within the sources. In addition, we will also present the possible effects on other observed CR properties such the Galactic diffuse $\gamma$-ray emission, s/p ratios and the anisotropies.

ACKNOWLEDGMENTS
The authors would like to thank the anonymous referee for his/her constructive comments.

REFERENCES
Abdo A. A. et al., 2010c, Sci, 327, 1103
Ackermann M. et al., 2010a, Phys. Rev. D, 82, 092003
Ackermann M. et al., 2010b, Phys. Rev. D, 82, 092004